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A STUDY OF NEGATIVE IMPEDANCES

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A THESIS

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Master of Science in Electrical Engineering

by

Henry Emerson Meadows, Jr.

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A STUDY OF NEGATIVE IMPEDANCES

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I wish to express my sincere appreciation to Professor M. A. Honnell for his suggestion of the problem for this study and for his invaluable guidance in its investigation.

SUMMARY

A negative impedance is a two-terminal device which can be considered to act as the algebraic negative of its corresponding positive (passive) impedance when connected into a network, provided the resulting circuit is stable. This thesis is concerned with the production of negative impedances, the experimental investigation of stability conditions for these impedances, and some applications for negative impedances.

A practical negative capacitance is described which approximates closely the behavior of a negative capacitance over a frequency range of about two decades. The most significant parasitic element associated with the negative capacitance is shown to be a variable negative resistance which remains constant over a limited frequency range. It is demonstrated that the negative capacitance can, with stability, be connected to a positive capacitance of equal magnitude if certain limitations are placed on the equivalent series resistance of the positive capacitance.

It is shown that a shunt-type negative capacitance circuit can be employed to compensate for the shunt capacitance of a pentode amplifier over a frequency range of about five to one. Moreover, the compensating amplifier provides some additional gain. The high-frequency response of the complete amplifier is shown to be little different from that of two pentode stages with shunt peaking; but

no inductances are required, and the very simple compensating stage introduces no low-frequency phase-shift due to screen-grid or cathode impedance.

A circuit for realizing in an active network a resistive impedance inversely proportional to the square of the frequency is seen to be inherently unstable in its ideal form, but stable in its practical form if certain restrictions are placed on the driving sources for the impedance. In the practical circuit investigated, the impedance possesses a small but non-zero reactive component, and the resistive component, even in theory, can approximate the desired inverse square frequency variation only in a limited frequency range.

A review of the literature concerning negative impedances is included in the thesis.

CHAPTER I

INTRODUCTION

Definition of the Problem.--Ordinary lumped linear passive impedances are the basic elements in most of the conventional electrical network theory as this theory is widely known today. The properties of these impedances taken individually are well known and may be expressed in a precise manner by the use of differential equations, complex algebra, or the transformation calculus. Furthermore, in connection with synthesis of impedances to effect prescribed results described in terms of one of these mathematical techniques, much has been learned about the general properties of arbitrary combinations of a finite number of these impedances. These general properties place theoretical restrictions on the possible character of any combination of the type of impedances mentioned.

This thesis deals with two-terminal impedances subject to a somewhat different set of restrictions, and of such character that they, for purposes of analysis by any of the mathematical treatments named, may be considered to be the algebraic negatives of corresponding impedances of the lumped passive type. (This equivalence will, however, even for small signals, be valid only upon the assumption of suitable characteristics by the network external to the "negative impedance", as will be seen later.)

In seeking a precise and useful definition of a negative impedance several approaches need to be made, since there are involved additional considerations not required by ordinary impedances.

In an elementary way, it might be said that a negative impedance is a two-terminal device which, for steady-state sinusoidal excitation, requires the (RMS) current entering one terminal to be proportional to the (RMS) voltage across its two terminals, but directed oppositely to the current flow which would occur if the device were replaced by its corresponding positive (passive) impedance. Thus the current flowing in a negative resistance experiences a rise in potential as it flows through the negative resistance.

Further, the power and energy characteristics of a negative impedance distinguish it from an ordinary impedance. A positive impedance either absorbs energy or stores energy. The only energy which can ever leave a passive impedance is energy previously stored. On the other hand a negative impedance either produces energy (so far as the external circuit is concerned) or lends energy. The only energy which can ever enter a negative impedance is energy previously lent to the external circuit. Thus the instantaneous power entering a negative inductance in steady-state sinusoidal operation varies at the double frequency but is at all times either zero or negative.

If a circuit containing negative impedance is analyzed by the usual a-c methods using complex algebra (still assuming stability), it can be shown that the frequency variation of both real and imaginary components is exactly like that for the corresponding positive impedance, except for a reversal in algebraic sign of both components.(1) This is extremely important, since it explains in part the usefulness of a pure negative element. Consider, for instance, a resistance in series with a constant inductance in some frequency range of interest. If it is desired to eliminate the reactive effect of this inductance a series capacitor can be employed, and the total reactance "tuned" to zero. This cancellation of reactances occurs, however, at only one frequency. But if a negative inductance equal in magnitude to the positive inductance (in the frequency range of interest) is connected in series with the positive inductance, the total impedance in this entire band of frequencies is purely resistive.

All of the statements so far have assumed that the circuit consisting of the negative impedance and what has been called the "external" impedance is stable. This assumption is similar and, in fact, closely related to the well-known statement that the gain of a single loop feedback amplifier is

$$A_f = \frac{A}{1-AB} \tag{1}$$

in the region of stability. It has been found that the stability of a negative impedance device is dependent on whether the device is "voltage-controlled" or "current-controlled". A voltage-controlled device produces a "shunt-type" negative impedance. Such an impedance is stable when connected to a positive impedance having a smaller magnitude at every frequency. A current-controlled device produces a "series-type" negative impedance.(2) This type of impedance is stable when connected to a positive impedance having a smaller magnitude at all frequencies. These conditions for stability are ordinarily sufficient but not necessary, as will be seen later.

A precise mathematical definition of a negative impedance in terms of the complex frequency variable, $s = \sigma + jw$, of the LaPlace Transform has been given by H. W. Bode.(3) The definition (paraphrased) says that $Z(s)$ is a negative impedance if

(1) $Z(s)$ is a ratio of polynomials in s (with real coefficients).

(2) $Z(s)$ has no poles for which $\sigma > 0$ (and only simple poles with negative residues on $\sigma = 0$).

(3) Re $\left\{Z(jw)\right\} \leq 0$ for all (real) w.

This definition, although not of primary importance in the work of this thesis, is nevertheless of considerable theoretical value.

It can be shown that the existence of a source of negative resistance, such as the dynatron, makes possible (at least mathematically) the production of any negative impedance.(1) A list of devices for the production of negative resistance, along with an excellent bibliography concerning these, has been given by E. W. Herold.(4) This review of these devices includes most of the earlier work on the subject of negative impedance. The negative impedances of real concern today are produced by controlled regenerative feedback in amplifiers. This method, although mentioned in Herold's list, did not become practical until stable wide-band amplifiers had been developed to their present reliability, and methods for prediction of stability in complex feedback circuits had become more fully known.

Some of the later advances involving negative impedances are the cathode-coupled negative resistance,(5,6) the continuing work in the analysis of general negative-impedance oscillators,(7) and the development of a practical negative-impedance repeater for telephone lines.(8,9)

Purpose of Research.--The research for this thesis had four primary objectives. These are as follows:

(1) The production of a negative capacitance free from excessive parasitic effects (either dissipative or anti-dissipative) over a considerable range of frequencies. Such a negative capacitance has varied possibilities of practical application such as cancelling unavoidable capacitances and use in special filter circuits.(10)

(2) The investigation of the practical use of a circuit proposed by Verman for producing a dissipative resistance proportional over some frequency range of interest to the inverse square of the frequency.(1) Such a circuit may be useful in producing filters with exceptionally good phase characteristics.

(3) The investigation of amplifier compensation at high frequencies by means of negative capacitance. Such compensation implies more uniform phase and frequency response than compensation by means of inductive effects. Particularly of interest was the problem of accomplishing this result without adding an undue number of vacuum tubes.

(4) The investigation into the stability of the circuits discussed above. This aspect of the research will not be discussed separately, but rather in connection with each circuit. It must be realized that in dealing with negative impedances the possibility of instability is never distant. This fact accounts for many of the practical difficulties in their use; but, just as with feedback amplifiers, the difficulties can ordinarily be overcome with sufficient analytical insight.

CHAPTER II

BRIEF HISTORY AND REVIEW OF THE LITERATURE

Negative Resistance.--The term "negative resistance" first arose in connection with the inverse voltage-current relations of arcs and certain glow discharges in gases. These devices are often relatively slow in operation, although the arc was used in early radio transmitter oscillators. Furthermore, because of their inherent nonlinearity and lack of reliability they are not of usefulness in the production of practical negative circuit elements. With the advent of the dynatron in 1918 the concept of negative resistance achieved great prominence. This development led to the use of negative-resistance oscillators as radio-frequency generators, and the use of dynatrons to measure the resonant impedance of tank circuits for oscillators and amplifiers. The problem of explaining in a quantitative manner the operation of the dynatron oscillator (and other nonlinear negative resistance oscillators) was attacked by Balth van der Pol(11) and others. The listing and review by E. W. Herold,(4) which has been mentioned above, outlines many of the early developments concerning negative resistances.

Negative-resistance oscillators have continued to receive attention since the advent of the dynatron. Cledo Brunetti has established a method of predicting the amplitude of oscillation which is both accurate and instructive.(12) His method is essentially the plotting of a curve of the magnitude of negative resistance versus the amplitude of the applied sinusoidal voltage. (This method involves the use of a negative-resistance bridge or a graphical operation on the static current-voltage characteristic.) This curve is then compared with the resonant impedance (L/CR) of the tank circuit, to predict the amplitude of oscillation, the possibility of amplitude instability, and the frequency range for which oscillation can be sustained (with variation of the capacitance, C). This technique is important because it avoids the necessity of solving a linear differential equation, whose coefficients are experimentally-determined polynomials (an approximation), in a series. At the same time this method lends greater insight into the phenomena produced by negative-resistance oscillators.

Newer sources of negative resistance of importance are the crystal diode,(13) biased (negatively) into the breakdown region, the transitron,(14-17) and the cathode-coupled negative resistance,(5,6) which operates to frequencies as high as 600 mc with a miniature duplex triode. The negatively-biased crystal diode is a negative resistance of the current-controlled type. Such a characteristic is also demonstrated by a properly-biased transistor. The cathode-coupled negative resistance is of importance not only because of its possibilities at very high frequencies, but also because it can possess a stable and more nearly linear characteristic than the dynatron. This property is of importance in the use of negative resistance to provide two-terminal amplification.

Even at present the difficulties presented by nonlinear differential equations have not been sufficiently resolved to permit a complete understanding of all the phenomena associated with negative-resistance oscillators in a reasonably simple manner. However some of the principle facts are now well known and may for instance, be demonstrated by means of admittance loci (with frequency as the running parameter).(18)

The introduction of the dynatron and its instances of practical application led to theoretical investigations by such men as A. C. Bartlett,(19) Balth van der Pol,(20) and L. C. Verman,(1) who described the possibilities for new types of impedances not restricted to the passive elements and their combinations. Consider the basic circuit shown in Fig. 1. The input impedance of this network, calculated in the usual manner of complex algebra (and assuming stability) is

$$Z_t = -m^2/Z \qquad (2)$$

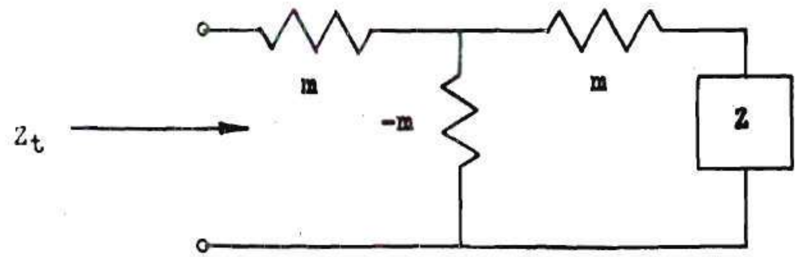
Now if the element m is a resistance, R, the element (-m) is a negative resistance, -R, and $Z = jwL$, the impedance of an ordinary inductance, then

$$Z_t = -R^2 jwL = 1/jwC' \qquad (3)$$

where $C' = -L/R^2$, a negative capacitance. On the other hand, if $Z = 1/jwC$, the impedance of an ordinary capacitance, as in Fig. 2, then

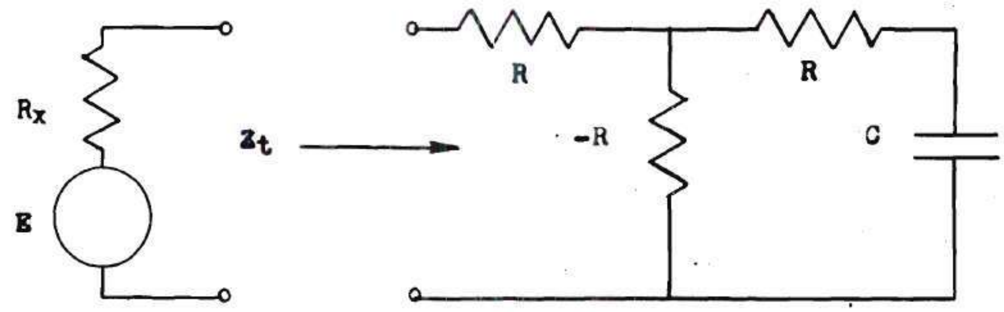
$$Z_t = -R^2/(1/jwC) = -jwR^2C = jwL' \qquad (4)$$

where $L' = -R^2C$, a negative inductance. It is also easily shown that the negative impedances thus obtained perform under transient conditions just as the algebraic negatives of positive impedances would perform



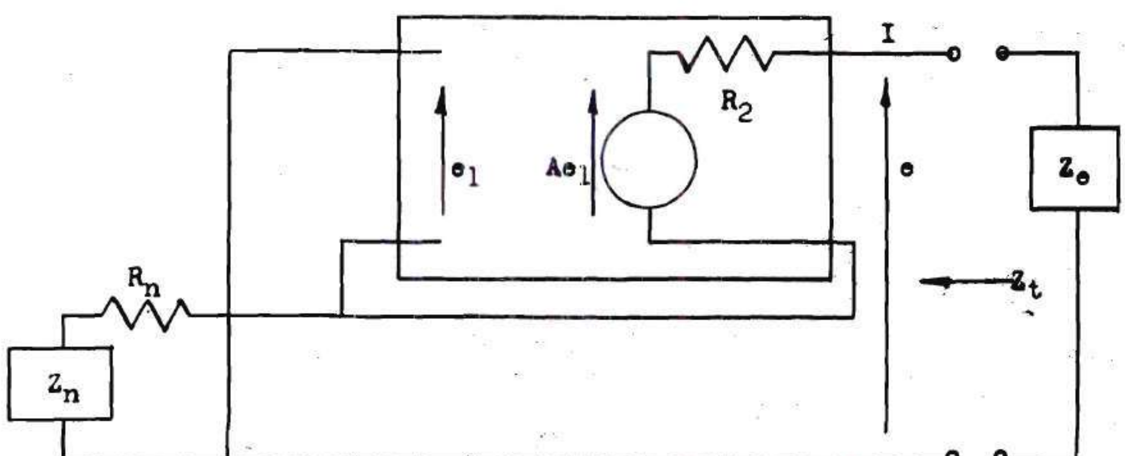
Theoretical Negative Impedance Circuit

Figure 1



Circuit for Negative Inductance

Figure 2



Series-Type Negative Impedance

Figure 3

according to the ordinary linear differential equations of electric circuit theory. It should be noted that these results depend on the validity of the circuit drawn to represent the physical facts. For instance if the resistances in the series arms are subject to skin effect, the results are no longer correct outside of a certain frequency range. In an exactly like manner the argument of the complex number representing the negative resistance (no matter how the negative resistance is obtained) can approximate $-\pi$ only in a certain frequency range, thus restricting the results given. This should cause no alarm, however, since there are practical limitations of frequency and amplitude of signal for almost all circuit elements, passive or otherwise.

After the production of a negative capacitance and inductance it is possible to assemble combinations to form negative impedances of arbitrary complexity (so long as some method of assuring stability is employed). However, a more interesting type of combination impedance results from two-terminal circuits containing both positive and negative elements. Such impedances have been termed "general" or "active" impedances and have been investigated to some extent by Bode.(3) The stability or instability of general active impedances in a network is not, as yet, fully predictable in the sense that necessary and sufficient conditions for stability are known in every case, although Bode lists rules which cover many useful and interesting cases. A particular set of these general active impedances was examined by Verman. This set, produced by the reinsertion of negative elements into the basic T of Fig. 1 consists of impedances proportional to $(jw)^n$, where n can be any integer. In this result, if n is even the resulting impedance is dissipative or antidissipative, while if n is odd the resulting impedance is non-dissipative. Many of these circuits would seem to be inherently unstable; but since, as noted before, the characteristics desired can at best be obtained only over a finite range of frequencies no matter what the desired results, the circuits may be modified to give stability. Such modifications must be controlled so as to minimize undesirable effects in the frequency range of interest. This problem is difficult, but not always impossible to solve. The modifications will, however, make certain of the theoretical results invalid, as will be seen later.

The circuit of Fig. 2 has been investigated experimentally. In this investigation, conducted by Cledo Brunetti and J. A. Walschmitt,(21) a transitron was used to supply the negative resistance required in the shunt branch. Since the transitron is a shunt-type negative resistance it would be expected that the resistance associated with a source such as shown in Fig. 2 would be restricted to values less than some definite maximum in order to insure stability. This limitation was borne out experimentally, and in fact, is characteristic of the type of limitation, which ordinarily restricts the use of negative impedances of all kinds. Subject to this restriction, however, it was possible to closely simulate a negative inductance over a frequency range greater than five octaves. A most important effect noted in this study was the extreme necessity for precision in circuit elements in order to obtain the theoretical result predicted from the network of Fig. 2. This precision is necessary because of the cancellations which must occur in the expression for Z_t in terms of the components of the network (see appendix). An extreme example of the result of small variations in the element values (in a rather unfavorable case) is reported by Brunetti and Walschmitt. In this example a 1.3 per cent variation of the negative resistance changes the power factor angle of Z_t at a frequency of 1000 cps from its ideal value of -90.0 degrees to -17.6 degrees.

General Negative Impedance Using Amplifier.--As noted above, any negative impedance can be produced by combinations of negative resistance, inductance, and capacitance obtained with Verman's circuits for negative elements if the final circuit can be made stable. Even assuming stability, however, this method of producing an arbitrary negative impedance is somewhat impractical owing to the difficulty of reducing the undesired residual effects (such as the parasitic series resistance in a negative inductance as described above).[1] These difficulties can be more easily resolved by using a different method for producing the negative impedance. This method involves the use of controlled positive feedback applied in an amplifier.

In 1931, circuits based on this method were shown by George Crisson(2) which can produce any desired negative impedance provided that the positive of the impedance can be constructed in the form of a two-terminal network. These circuits, shown in Fig. 3 and 4, Crisson ascribed to R. C. Mathes and H. W. Dudley respectively. The ideal circuits drawn require amplifiers with no phase-shift, no polarity reversal from input to output, a constant gain (greater than unity),

[1]An arbitrary negative impedance can also be produced by constructing the inverse of the positive impedance, and inserting this in place of the capacitance in Fig. 2. This method was not investigated in the research for this thesis.

an infinite input impedance, and a constant resistive output impedance at all frequencies. If these requirements are fulfilled and the proper constraints are placed on R_n and R_3 (see appendix) the resulting impedances seen by the voltage e, are $Z_t = Z_n(A - 1)$ for Fig. 3 and $Z_t = -Z_n/(A - 1)$ for Fig. 4. Because of the method of connecting the input and output terminals of the amplifier in each case, Crisson called the negative impedance produced in Fig. 3 a series-type negative impedance and that of Fig. 4 a shunt-type negative impedance.

In his investigation of series-type and shunt-type negative resistances produced by this method (in this case a finite resistive input impedance is not detrimental) Crisson established the following basic rule concerning the stability of a negative resistance inserted in a purely resistive circuit:

> . . . a negative resistance of any desired value may be inserted in a circuit having any positive resistance R provided that the inserted resistance has the characteristics of the series type when the inserted negative resistance is numerically smaller than the positive resistance or the characteristics of the shunt type when the negative resistance is numerically larger than the positive resistance.(2)

It will be noted that this is essentially the same criterion for stability as given without proof in the introduction if the word resistance is replaced by impedance and if the required inequalities are stipulated to be true at all frequencies. However, to obtain a more delicate test for stability, the Nyquist plot of feedback loop transmission factor may be examined. If an external impedance Z_e (assumed here to be a positive impedance) is connected to Z_t in either the series or shunt-type circuit, the result is a single-loop feedback amplifier of the proper type for Nyquist's Criteria to apply. In fact the resulting feedback amplifier is not different in the two cases except that the feedback impedance and the external impedance are interchanged. Accordingly the stability criteria concerning the impedances are essentially the same. Peterson, Kreer, and Ware, in their classic article entitled "Regeneration Theory and Experiment"(22) show that the Nyquist Criteria may be paraphrased for the series-type circuit to involve plotting the locus of the complex ratio

$$L_1 = \frac{-Z_t}{Z_e} \tag{5}$$

and for the shunt-type circuit, the locus of

$$L_2 = \frac{Z_e}{-Z_t} \tag{6}$$

In each case, if the locus (plotted for all real frequencies) encloses the point 1 + j0 the circuit is unstable, and conversely, if the locus excludes the point 1 + j0 the circuit is stable (either absolutely or "Nyquist stable").

Criteria for stability of combinations of negative impedances and general impedances also have been developed by H. W. Bode(3) to cover many cases of practical interest. These include conditions for stability and instability of connections of both shunt-type and series-type negative impedances and their combinations.

Experimental work on negative capacitance and resistance circuits has been carried on by Cledo Brunnetti and Leighton Greenough.(10,23) Their results are probably the most useful practical results shown in the literature to date. They used a feedback amplifier of two stages with a shunt-type circuit. The results reported however are concerned primarily with the relation of the amplifier parameters to the negative impedance, rather than the frequency characteristics of the negative impedance. A theoretical investigation of the effects of parasitic capacitances and interstage coupling networks has been published by E. L. Ginzton.(24) The investigation considers the case of a two-stage resistance-coupled amplifier only, but results are indicative of probable results with more complex circuits. Ginzton also shows practical examples of the use of negative impedances in circuit design. These examples, along with similar examples shown by Terman, Buss, Hewlett, and Cahill(25) in connection with benefits of negative feedback include cancellation of power-supply impedance, improving the selectivity of intermediate-frequency amplifiers, and increasing the maximum modulation index (for no distortion) of a diode detector.

Recent developments in the field of negative impedance include the cathode-coupled negative resistance mentioned earlier, possibilities for battery-powered, self-enclosed negative impedances employing low-drain transistor amplifiers, and the use of a negative-impedance repeater designed for application to long local telephone loops.

CHAPTER III

PRODUCTION OF A SHUNT-TYPE NEGATIVE CAPACITANCE WITH SMALL PARASITIC EFFECTS

A shunt-type negative capacitance can be of much use in many specialized problems. An example is the cancellation of an undesired parasitic capacitance in a measuring device such as a bridge, Q meter, oscilloscope, or vacuum tube voltmeter(10). In such applications an adjustable negative capacitance with good frequency characteristics and a negligible power factor could be a valuable device. Other possible applications in the field of network synthesis may become important. (As pointed out by Bode,(3) any transfer relation between two sections of a network which is obtained through the use of active elements in the network can also be obtained with passive elements and amplification if the network employing active elements is stable. Nevertheless the use of active elements may at times be preferable in cases where such considerations as residual losses, circuit unbalance, stray capacities, or extremely low or high impedance levels are controlling factors in a design.)

Amplifier Requirements.--The realization of a useful negative capacitance is essentially a problem of amplifier design in which the amplifier characteristics must be carefully controlled. The amplifier requirements discussed below are the primary considerations in a proper design. They are also important in the amplifier compensation problem to be discussed later.

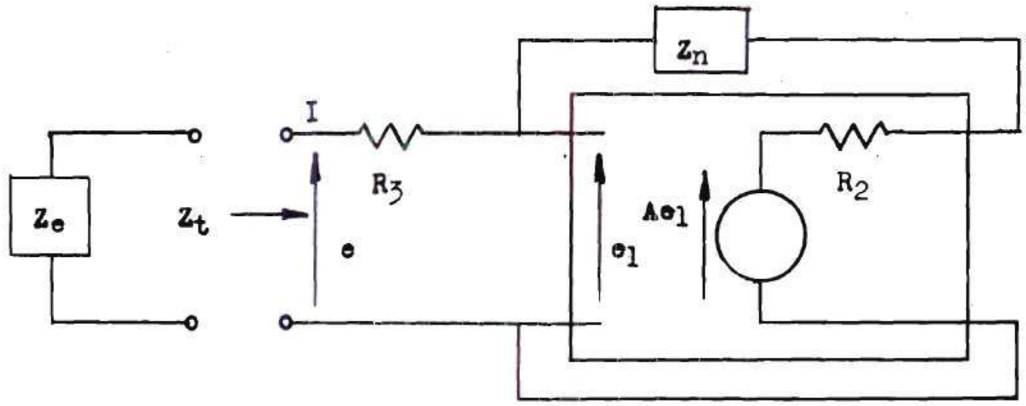
In the shunt-type negative impedance circuit of Fig. 5, the negative capacitance is easily shown to be

$$-C_n = -C_f(A-1) \tag{7}$$

This capacitance is ordinarily (in the mid-frequency region) in series with a negative resistance

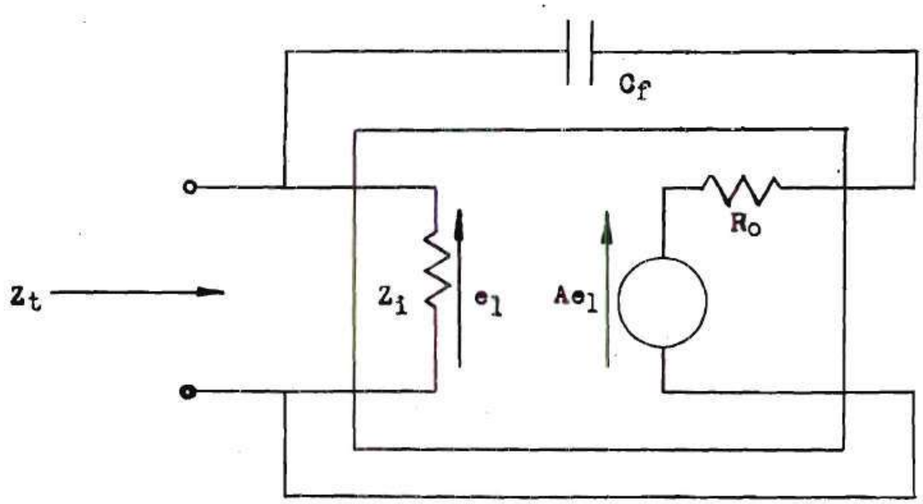
$$-R_n = -R_o/(A-1) \tag{8}$$

where R_o is the output impedance of the amplifier; thus the equivalent circuit is Fig. 6. This negative resistance could be cancelled to any required degree with a positive resistance, but this is not ordinarily done, since doing so leads to instability in many applications. Yet if the loss angle of the negative capacitance is to be small, R_n must not be large. For values of A greater than about five, the magnitude of negative resistance is approximately inversely proportional to A. Thus a larger A is desirable in order to reduce R_n. (This comment assumes that R_o is not increased to increase A, as would generally be the case if the gain were increased in some stage other than the last in an amplifier without overall voltage-controlled feedback.) However, a value of A which is greater than about 12 will increase unduly the possibilities of oscillation in the amplifier. This statement applies particularly to feedback amplifiers, and it has been noted before that the entire negative-impedance circuit to be employed is a feedback amplifier even if no additional nominally negative feedback is employed in stabilizing the amplification A. Furthermore it must be remembered



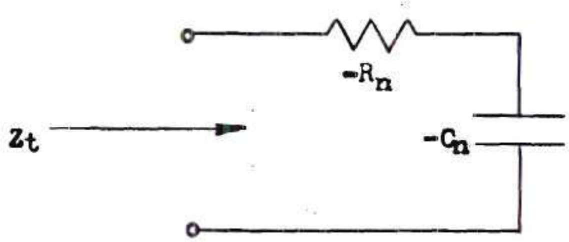
Shunt-Type Negative Impedance

Figure 4



Shunt-Type Negative Capacitance

Figure 5



Equivalent Circuit of Negative Capacitance

Figure 6

that C_f is approximately inversely proportional to A for large A and a fixed value of C_n. Thus if C_n is small C_f may be "swamped out" by parasitic elements. For example if $C_n = 100$ uuf is desired, then with $A = 11$, the feedback capacitor is only $C_f = 10$ uuf. A capacitor of such size with very good characteristics is not difficult to procure, but the undesired stray capacities across C_f, and in fact across R_o and other circuit elements in the amplifier may in this case be of the same order of magnitude as C_f. This obviously renders the circuit of Fig. 6 inappropriate, and the resultant impedance, Z_t, may differ greatly from that of a negative capacitance. The negative impedance Z_t may also be changed from its ideal form by virtue of the fact that $|Z_i|$ is no longer large enough to be considered infinite in comparison with $1/wC_f$ in the useful frequency range. From these considerations, then, it seems that a reasonably small value of A is most practical.

There is also a lower limit for A. If A is only slightly greater than one, the resulting impedance will be negative in character, but for $A \doteq 1$, small fractional variations in A will produce extremely large fractional variations in Z_t. This is readily seen to be undesirable. A compromise range of amplifications to suit most of these requirements would seem to be

$$2 \leq A \leq 12. \qquad (9)$$

The limits here are extremes which are valid for most practical values of negative capacitance in the range of frequencies where resistance-coupled amplifiers are most useful (to perhaps 100 kc or 200 kc).

In general, the output impedance should be resistive in the frequency range of interest, and its magnitude should be as small as possible. One method of producing a very small output impedance is to employ overall voltage-controlled feedback. This method has the further advantage of producing the other well-known benefits of negative feedback.

Phase-shift in the amplifier should be held to an absolute minimum in the frequency range of interest. If this is assured, then the amplification will ordinarily be constant. Phase-shift tends (at least as a first approximation) to change the equivalent circuit of Fig. 6 to that of Fig. 7. The shunt resistance $R(w)$ may be considered infinite in the mid-frequency region. In general, for resistance-coupled amplifiers (with or without feedback), above this region $R(w)$ assumes negative values which decrease in magnitude as w increases. At frequencies below this region $R(w)$ is positive, and decreases as w decreases.[2] This circuit represents the character of Z_t reasonably well so long as the amplification does not change appreciably. Outside of the region of substantially constant amplification the phase-shift in the amplifier is so great that Z_t is no longer even approximately of the form $1/(-jwC_n)$, and it is usually necessary at these frequencies to insure stability, rather than to attempt the correction of Z_t. If sufficient care is taken in the design, and sufficient control is exercised over the phase-shift

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[2]An exact solution for the proper frequency variation of $R(w)$ (but expressed in a different form) for the case of two identical resistance-coupled amplifier stages in cascade is shown by Ginzton(24). In the general case the solution will be somewhat similar and may be represented as we have shown only under the conditions stated. In any stage employing inductive peaking the statements concerning $R(w)$ would need modification. These statements concerning $R(w)$ were verified for the feedback amplifier circuit described below.

outside the mid-frequency region, any desired frequency range can be obtained (at least in theory) in which Z_t does not deviate appreciably from its ideal form.

It is possible that phase-shift in the amplifier could be introduced in such a way as to effectively reduce the loss angle of the negative capacitance. This effect would be possible over a limited frequency range, and its mechanism would involve simply the variation of $R(w)$ in the proper fashion to dissipate the power entering the network through $-R_n$. This compensation would probably involve some type of inductive peaking or perhaps some more refined form of interstage-coupling network with a phase shift expressly designed to provide the parameter (A-1) with the proper phase-shift approximately to cancel the loss angle of Z_t. This type of compensation might allow the use of an amplifier with a somewhat larger output impedance. It should be noted that stability of such a compensated circuit cannot be taken for granted. This aspect has not been investigated.

Nominally negative feedback can produce many of the properties desired in the amplifier. In the frequency range where the magnitude of loop transmission coefficient $|AB|$ is much greater than unity, all the customary benefits of negative feedback apply. These of course include greater constancy of amplifier parameters with respect to power supply voltage variations and a better approach to linearity. Greater constancy of amplifier parameters makes prediction of stability simpler, since the methods of prediction based on ideal networks can then be assumed to apply.

However, feedback sufficiently large to accomplish these results can itself introduce instability at frequencies where AB is changing rapidly. This phenomenon is well known, and methods are available to produce absolutely stable feedback amplifiers of the single loop variety. Use of the amplifier in a negative impedance circuit complicates these methods since the feedback is no longer of the single loop variety. This fact becomes especially important if individual stages possess internal feedback (such as cathode degeneration). In general, then, it is not easily possible to predict the stability of a negative impedance circuit if the amplifier concerned has overall negative feedback and internal feedback loops.

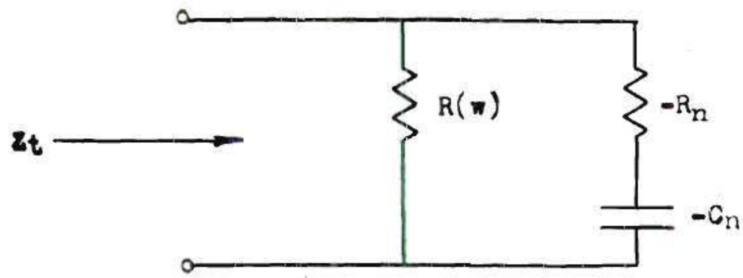
Overall feedback of the voltage-controlled type has several advantages in the construction of the desired amplifier. These include the usual benefits of negative feedback and the property of output impedance reduction(26). Such reduction provides a very practical way to reduce the parasitic resistance R_n.

Negative feedback enhances, also, the phase response of the amplifier. This property causes R(w) in Fig. 7 to be substantially infinite in the frequency range where

$$|AB| \gg 1. \tag{10}$$

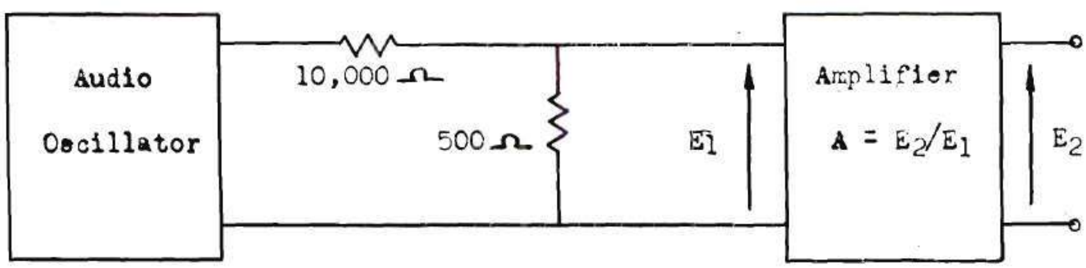
Naturally, R(w) will change more rapidly outside of this range than without negative feedback.

The value of amplification, A, must be positive and greater than unity in the circuits of Fig. 3 and Fig. 4. This implies not only a magnitude greater than unity, but also no phase reversal from input to



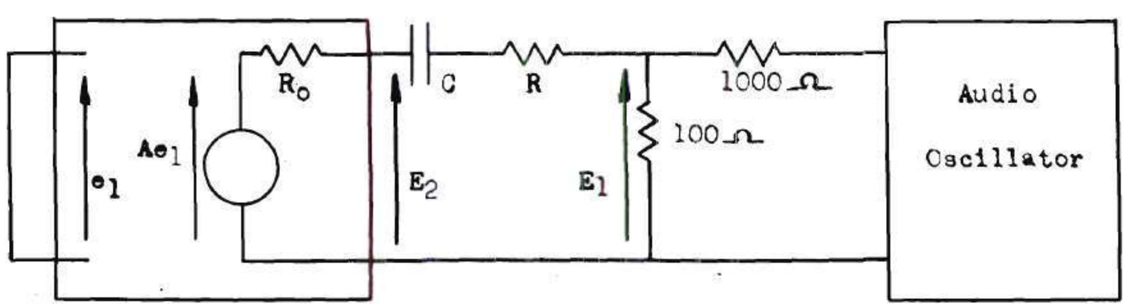
Effect of Amplifier Phase-Shift on Negative Capacitance

Figure 7



Measurement of Amplification

Figure 8



First Method of Measuring Amplifier Output Impedance

Figure 9

output. If all stages of the amplifier are grounded-cathode stages, then an even number of them are needed. Grounded-plate (cathode-follower) and grounded-grid stages introduce no phase reversal. It is important to note that each stage added introduces the phase-shift of its inter-stage network, thus contributing to the instability problem at high and low frequencies if feedback is employed, and to reduction of the magnitude of R(w) outside the mid-frequency region whether or not feedback is employed.

In general all two-terminal impedances which are considered linear for purposes of analysis become nonlinear impedances if they are required to accept too great an input voltage (or current). In the case of a negative impedance the tolerable input voltage is limited by the linearity of the amplifier parameters. The linearity of the amplifier can be improved by negative feedback. If this feedback is coupled with additional stages to provide a larger available output power, the negative capacitance produced can be made to allow an arbitrary maximum voltage, regardless of the value of capacitance (at least in theory).

Another method of achieving greater linearity is through the use of balanced push-pull circuits. Such circuits have the desirable property of cancelling even harmonic components of distortion introduced in the vacuum tubes. This method would probably be of advantage in applying a shunt-type negative capacitance to a circuit balanced with respect to ground. It would also make arbitrary the number of stages required since the capacitive feedback could be applied across the symetrical halves of the circuit in the case of an odd number of stages.

<u>Circuits Employed</u>.--Several amplifier circuits were employed during the investigation. They were evaluated according to the requirements discussed above, many of which became evident during measurements on the amplifier circuits used in negative-capacitance circuits. Two of these amplifiers are discussed below.

The gain of each amplifier was determined by standard methods using an audio oscillator, input attenuator for isolation, and vacuum-tube voltmeters as in Fig. 8. The determination was made over a range of frequencies to determine either the complete frequency response or the frequency response up to about 200 kc.

The output impedance of each amplifier tested could be expected to be essentially resistive in the mid-frequency range. Their measurement then was accomplished by use of the voltage division rule as illustrated in Fig. 9 or by means of the technique of Fig. 10. A frequency of 1000 cps was used for most of these measurements, although checks were made at other frequencies to insure reliability of the measurements. The blocking condenser, C, was chosen large enough to present negligible reactance compared to R. In using the voltage division rule, a non-inductive resistance R, was selected so that $E_2 \doteq \frac{1}{2}E$, (to give $R_o \doteq R$). Then the exact relation is

$$R_o = RE_2/(E_1 - E_2) \tag{11}$$

With the technique of Fig. 10 a constant voltage, e_1, was applied to the input of the amplifier and $E = Ae_1 = E_1$ was measured. Then the resistance, R, was shunted across the amplifier and $E = E_2$ was measured.

R was chosen so that $E_2 \doteq \frac{1}{2}E_1$ (to give $R_o \doteq R$). Then the exact relation is

$$R_o = R(E_1 - E_2)/E_2 \tag{12}$$

The circuit used for measuring negative capacitance and negative resistance is shown in Fig. 11.[3] It is a useful circuit because it can be reasonably accurate while maintaining stability. The bridge shown inside the broken line is a Western Electric 4A impedance bridge. The elements R_s and C_s are the standard elements in the bridge. These were used to provide a rough balance. The fine balance was provided by R_p, a precision decade resistance, and C_p, composed of a precision decade capacitance and a vernier capacitance with an error less than one uuf (neglecting external leads). These instruments were all General Radio Company standard elements. The input was maintained small enough during the measurements to allow no appreciable nonlinearity in the negative capacitance. However, near balance the small nonlinearities introduced by all of the circuit elements became appreciable in proportion to the bridge output. Furthermore hum and noise from the electronic instruments and the power line obscured the bridge output. For these reasons it was often found necessary to resort to the use of a filter in the detector circuit. The filter used was a variable filter covering most of the audio frequency range. The vacuum-tube voltmeter was used both as a null detector and as an amplifier to demonstrate the bridge output

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[3]This method is in principle like that used by Brunetti and Greenough(10).

on the cathode-ray oscilloscope. In the audio-frequency range headphones were also found useful in the detection of a precise null.

The method of measurement consisted of two balances. The bridge was first balanced with the negative capacitance connected at X. The parallel elements C and R were adjusted to provide stability. For this circuit stability will ordinarily be insured if $C_p \quad 2C_n$ regardless of the values of C_s, R_s, and R_p. It was also found helpful in some cases to use a low-impedance input circuit, such as provided by the inverted-L attenuator shown in Fig. 11. R_s and C_s were so adjusted as to give a balance sensitive with respect to variations of C_p and R_p. Upon making the fine balance the values $C_p = C_{p1}$, $R_p = R_{p1}$, and the frequency were recorded. Then the negative capacitance was removed by breaking the circuit at Y, and the bridge again balanced. This balance was made by variation of C_p and R_p only. The values $C_p = C_{p2}$ and $R_p = R_{p2}$ were noted, and the following formulas were employed to obtain R_n and C_n:

$$C_e = -(C_{p1}-C_{p2}) \tag{13}$$

$$R_e = \frac{-R_{p1}\ R_{p2}}{R_{p2}-R_{p1}} \tag{14}$$

$$-C_n = C_e \frac{1 + (WC_eR_e)^2}{(WC_eR_e)^2} \tag{15}$$

$$-R_n = \frac{R_e}{1 + (WC_eR_e)^2} \tag{16}$$

The method used is capable of reasonable accuracy if a proper adjustment of the independent variables R_s and C_s is made. This adjustment varies widely with frequency and was generally obtained by trial and error.

The cathode-coupled amplifier(5,6,26-29), shown in Fig. 12, has a mid-frequency voltage amplification[4] of

$$A_m = \frac{u_1(u_2 + 1)R_LR_k}{r_{p1}r_{p2} + r_{p1}R_L + r_{p1}R_k(u_2 + 1) + R_k(u_1 + 1)(R_L + r_{p2})} \tag{17}$$

and a resistive output impedance in the mid-frequency region of R_a, where

$$\frac{1}{R_a} = \frac{1}{R_L} + \frac{1 - \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{1}{g_{m1} + g_{m2} + 1/M}\right)}{r_{p2}} \tag{18}$$

and

$$\frac{1}{M} = \frac{1}{R_{p1}} + \frac{1}{r_{p2}} + \frac{1}{R_k} \tag{19}$$

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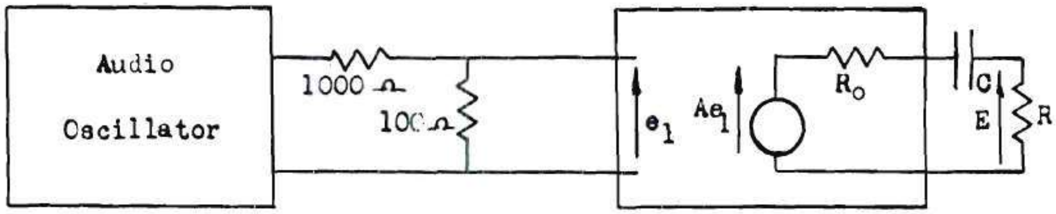
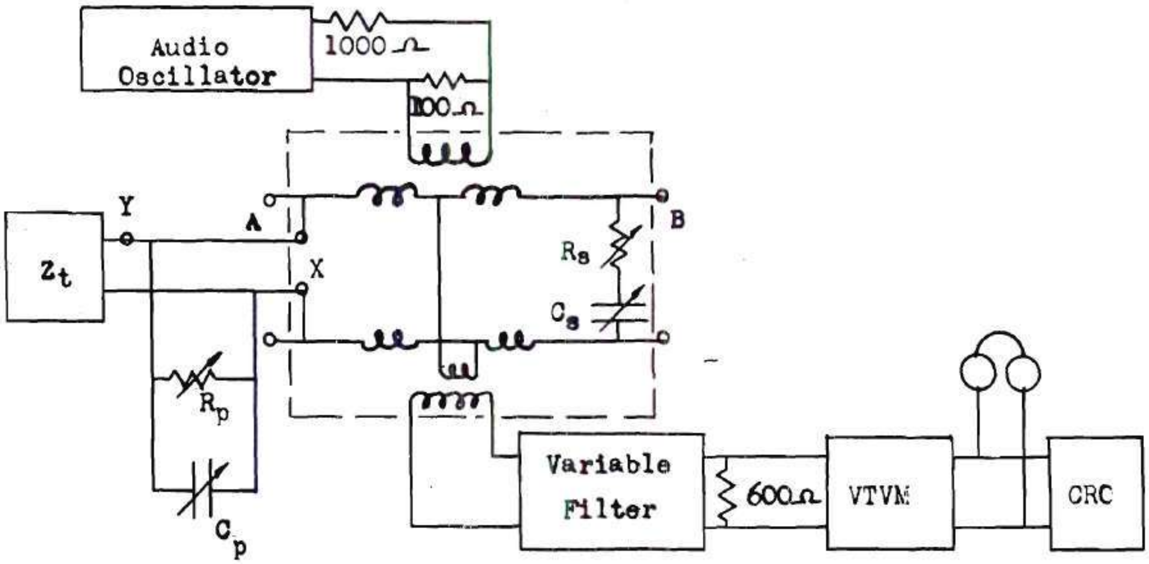
[4]See appendix.



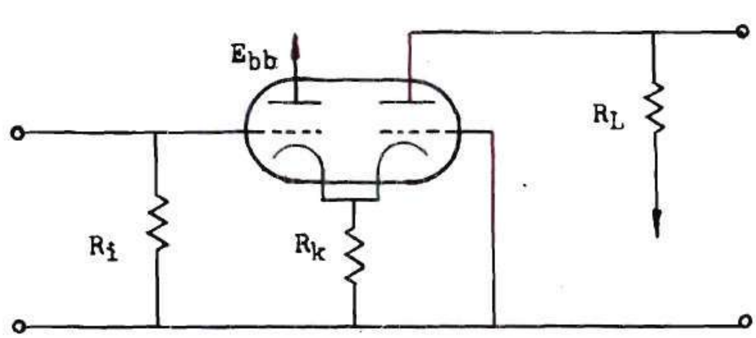
Second Method of Measuring Amplifier Output Impedance

Figure 10



Circuit for Determining C_n and R_n

Figure 11



Cathode-Coupled Amplifier

Figure 12

Its inherent degeneration makes its use at high frequencies very practical. Furthermore, it is direct coupled, and therefore introduces no low-frequency phase-shift. Its gain can be adjusted within the range mentioned above, and its output impedance can be made reasonably low using triodes with large transconductance.

A cathode-coupled amplifier was designed and built to provide a voltage amplification of two and an output impedance of about 1800 ohms. The circuit used was that of Fig. 12, and the duplex triode employed was a 6J6. The sections of this duplex triode have large ratios of transconductance to interelectrode capacitances, thus providing the possibility of a wide-band negative capacitance. However, the 6J6 tubes are miniature in size, and the voltage input which can be employed without clipping of either or both peaks was found to be quite small. In most cases an input voltage of one volt or more produced a decidedly distorted output waveform. For input signals as large as two volts the amplifier clipped noticeably.

Other characteristics of this amplifier were also somewhat unfavorable. Due to the low-frequency characteristics of the amplifier it was necessary always to maintain a very low-impedance path from the input to ground in order to prevent low frequency signals (in particular, 60 cps hum from power circuits) from obscuring the results desired.

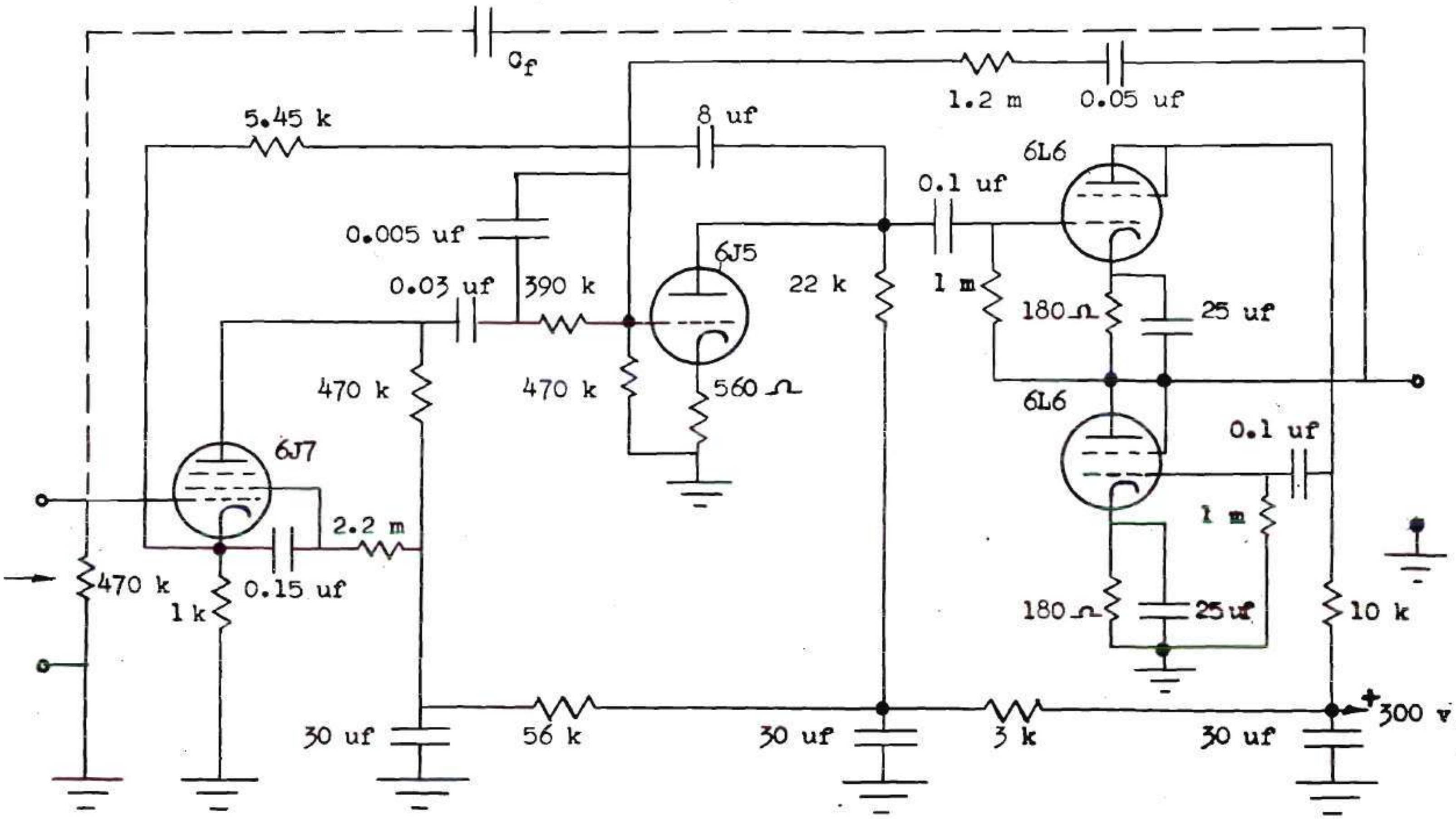
Although these characteristics of the amplifier were not desirable, a negative capacitance was constructed using a feedback capacitor of $C_f = 0.125$ uf. The resulting negative impedance was then experimentally determined to consist of a -1700 ohm resistance in series with a capacitance of -0.130 uf. These figures remained substantially constant over the audio-frequency range. By increasing the amplification, it would have been possible to obtain a capacitance multiplication; that is, the same value of C_n could be obtained with a smaller feedback capacitor. However, increasing the gain will not materially decrease the series resistance R_n, since R_n is approximately proportional to both the output impedance R_o and $1/A$, the reciprocal amplification, and R_o is approximately proportional to A. It can be seen that the loss angle of this negative capacitance is unreasonably large at most audio frequencies. This is not of too great concern since use of the same amplifier to obtain a smaller negative capacitance, say 0.001 uf, would result in the same series negative resistance and therefore a considerably smaller loss angle (less than one degree at 1000 cps). Nevertheless it was evident that, at least for experimental purposes, a more complicated amplifier circuit would be needed.

Many of the disadvantages of the circuit just discussed could be overcome by the application of a large amount of negative feedback applied in the voltage-controlled manner over two or more stages. In fact a feedback amplifier of this type fulfills most of the amplifier requirements listed above. The primary drawback to this approach is the complicated stability problem which it introduces.

The amplifier finally selected is a rather complicated, four-tube amplifier with several feedback paths. It consists of a conventional two tube resistance-coupled feedback amplifier in cascade with a double cathode-follower output stage. There is, in addition, a voltage-controlled feedback path from the output of the double cathode-follower to the grid of the second tube. The especially useful property of the double cathode-follower in this application is its exceptionally low output impedance. The particular feedback methods employed were necessary to avoid low-frequency oscillations resulting from large phase-shifts due to coupling capacitors.

The complete circuit, including element values of the feedback amplifier, is shown in Fig. 13. The voltage amplification of this circuit was, very closely, $A = 8.3$, and the output impedance $R_o \doteq 35$ ohms. This value for the output impedance is approximate; the exact value was evidently a relatively sensitive increasing function of the amplitude of input voltage. These parameters yield a theoretical magnitude of residual negative resistance (in the mid-frequency region) of about five ohms. The equivalent series resistance of the feedback capacitor, C_f, divided by 7.3, must also be added to this resistance to obtain the actual mid-frequency value. An equivalent magnitude of series resistance of the order of five ohms is not excessive, and a proportionality factor of seven between C_f and C_n is in the desirable range. These facts, coupled with the linearity of the feedback amplifier and its constancy of gain, indicated that the circuit was suitable for further experimentation to examine the properties of negative capacitance.

Negative Capacitance and Its Properties.--Two values of negative capacitance were produced using the feedback amplifier just discussed. The feedback capacitor was in each case selected from a group of various types and brands, the criterion being the magnitude of equivalent series loss resistance. For the first capacitance a Cornell-Dubilier capacitor



Feedback Amplifier Used to Produce Negative Capacitance

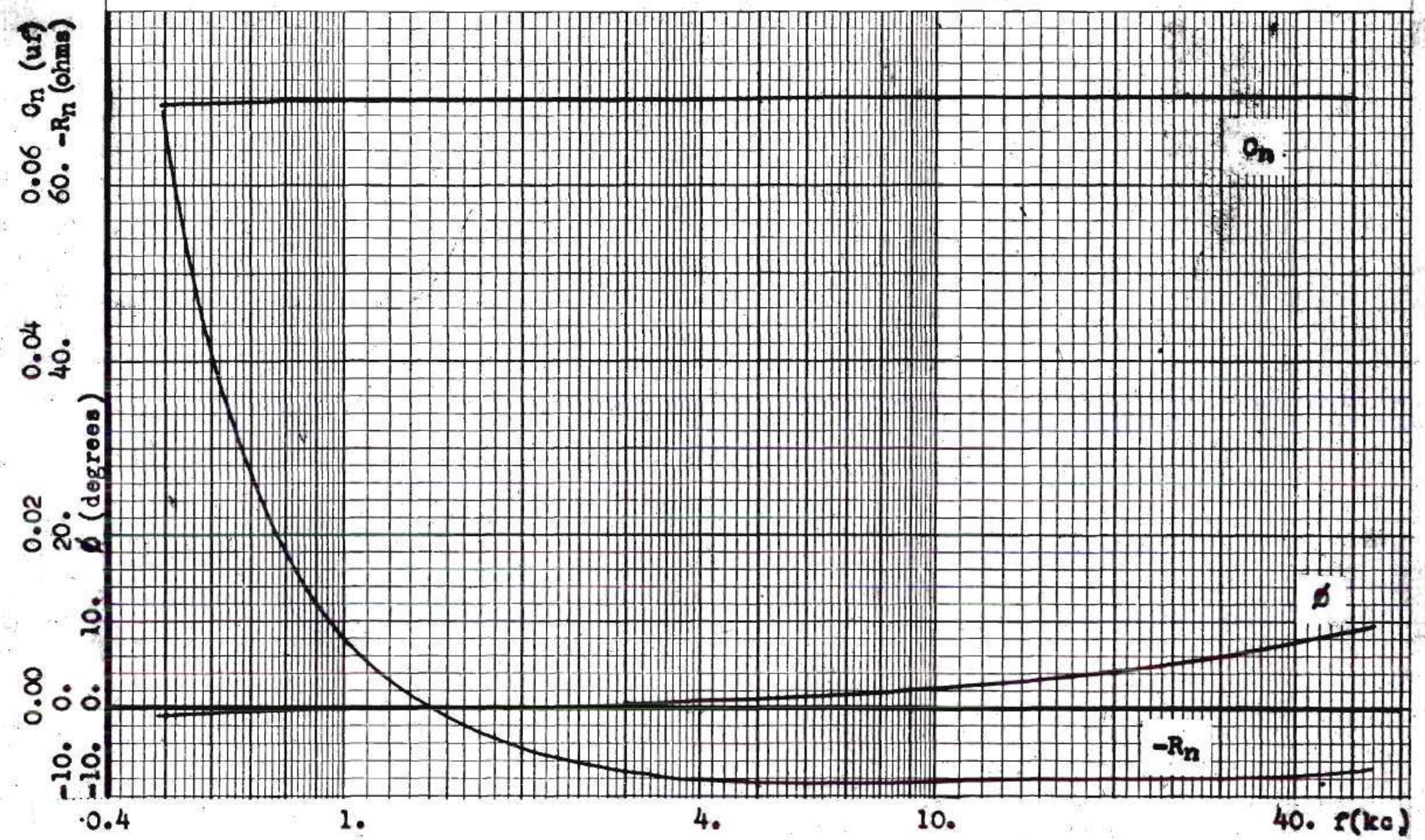
Figure 13

(nominally rated at 0.01 uf) with series values of $C = 0.00954$ uf and $R = 5$ ohms was chosen. The second capacitance chosen was a Western Electric capacitor (nominally 0.001 uf) with a measured capacitance of $C = 0.00101$ uf and a dissipation factor too small to be exactly determined with the General Radio Company 650-A impedance bridge at a frequency of 1000 cps. This dissipation factor was evidently in the order of 0.001, and the series resistance can probably be considered to be in the range 200 to 300 ohms.

In general, any physical element deviates from the ideal over part of the useful frequency spectrum. For instance the equivalent series inductance of a physical coil with small losses will increase as its first self-resonance (with its distributed capacity) is approached. The negative capacitance produced by an amplifier will similarly have minor variations with respect to frequency, and its residual series resistance will experience larger proportional variations. This variation is not primarily due to variation in magnitude of amplification, but to the accompanying phase-shift. Another cause for variation of the series negative resistance is the variation of equivalent series resistance of the capacitor.

The exact variation of C_n and R_n with respect to frequency is shown by tables 3 and 4 in the appendix. The most important results are shown graphically in Fig. 14 (for $C_f \doteq 0.01$ uf) and Fig. 15 ($C_f \doteq 0.001$ uf).

In the first case the predicted value of negative capacitance is $-C_n = -C_f(A - 1) = -0.069$ uf, and the predicted mid-frequency value of R_n is $-R_{no} = -(R_o + 5)/(A - 1) = -6$ ohms. The experimentally observed values of C_n ranged from 0.0690 uf to 0.0701 uf for frequencies between 500 cps



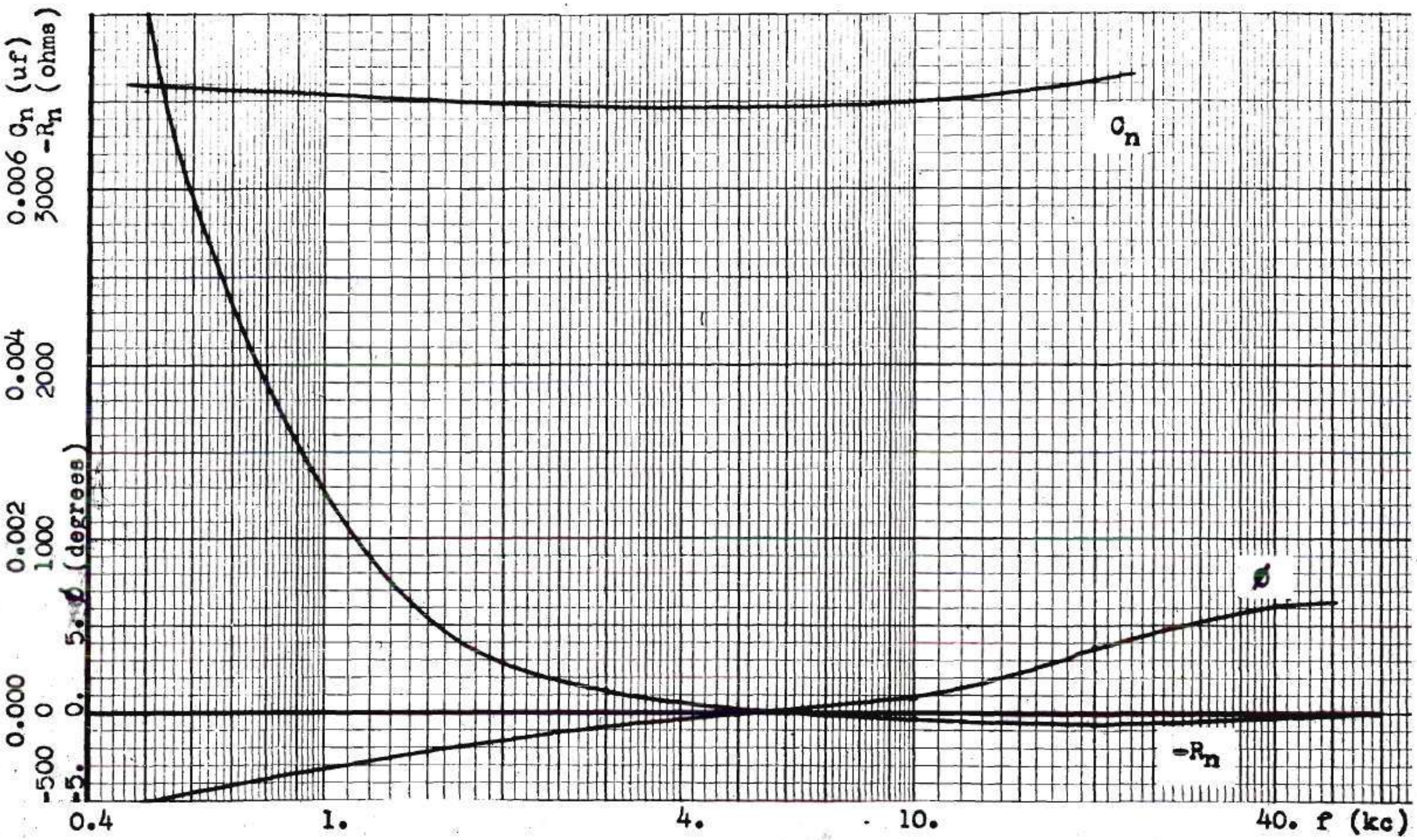
Frequency Characteristics of Negative Capacitance ($C_f \doteq 0.01$ uf)

Figure 14

and 50 kc. These figures represent a variation of 1.6 per cent over a frequency range of two decades. The variation of R_n over the same two decades conforms to the statements made concerning the approximate equivalent circuit of Fig. 7. In the mid-frequency region R_n is approximately eight ohms, a value which agrees reasonably with that predicted above. At lower frequencies R_n becomes negative (the actual resistance, $-R_n$ becomes positive), tending to increase in magnitude very rapidly at frequencies below 1000 cps.

Using the second feedback capacitor the predicted value of negative capacitance is $-C_n = -0.0073$ uf, and the predicted value of series resistance is in the range -31 to -45 ohms. The measured values of C_n differed somewhat from this prediction; the extreme values for C_n in this case, for the same two decades as considered above were 0.00691 uf and 0.01 uf. The latter figure was taken at a frequency of 50 kc, and the determination could not be made with the same precision as at lower frequencies. It is therefore reasonable to assume that the value $C_n = 0.0072$ uf, taken at 20 kc, is close to the exact extreme value, and upon this assumption the percentage variation of C_n from 500 cps to 50 kc is approximately 4.2 per cent. This larger variation is probably a result of inaccuracy in measuring a smaller value of C_n, rather than a greater variation of the true negative capacitance. The mid-frequency value of R_n again verifies the predicted result with sufficient accuracy, since the series resistance of the feedback capacitor was not accurately known.

Figs. 14 and 15 show also the variation of the loss angle of the negative capacitance with respect to frequency, and with actual losses a function of negative angles. The loss angle at most frequencies shown is



Frequency Characteristics of Negative Capacitance (C_f = 0.01 uf)

Figure 15

small enough to allow the negative capacitance to be considered a pure element for many purposes. The maximum magnitude of loss angle found was 9.0 degrees or 0.16 radians. At most frequencies the loss angle was less in magnitude than 0.1 radian, the value often used as a dividing line between pure elements and complex elements for engineering purposes. The power factor of these two negative capacitances is therefore small enough in magnitude to be comparable with that of a passive circuit element.

A graphical method of demonstrating the frequency characteristics of a negative capacitance is to plot the locus of the quantity S, defined by

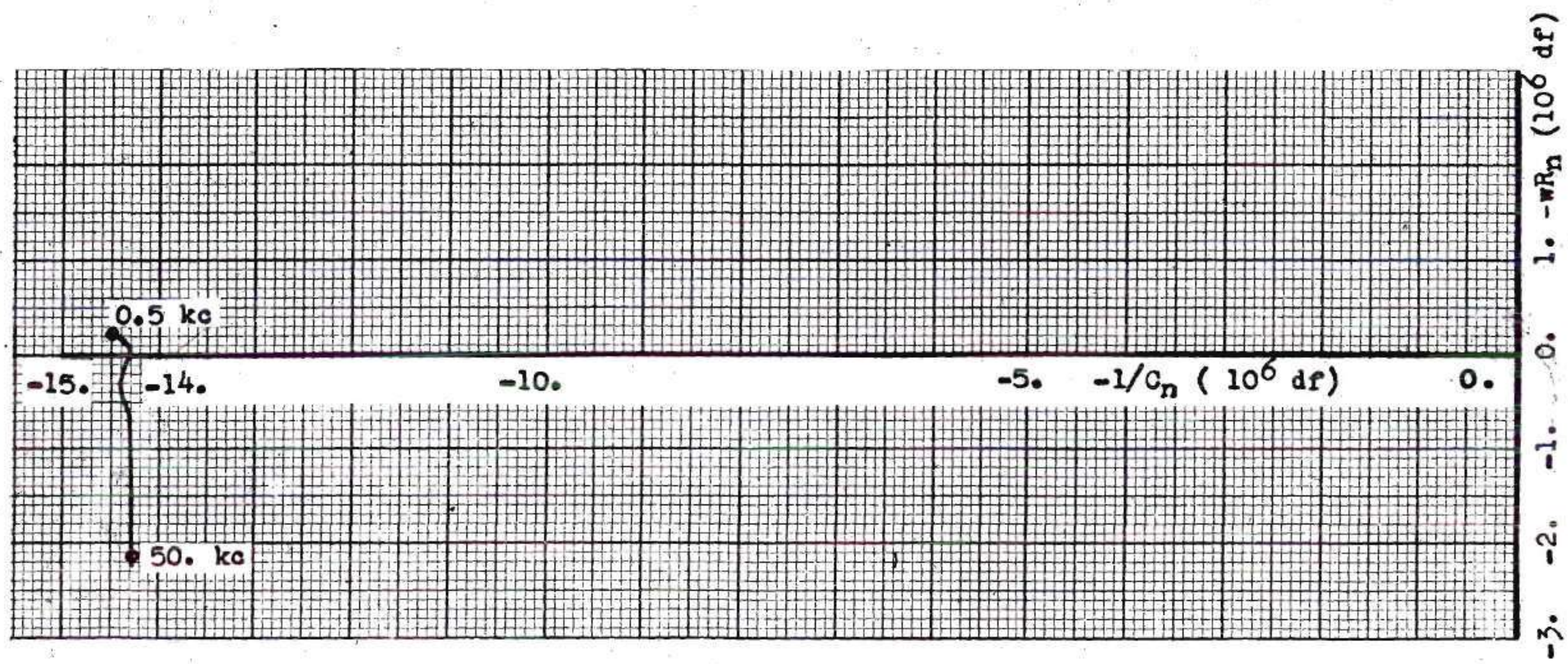
$$S = jwZ_t(jw) \tag{20}$$

where $Z_t(jw)$ is the negative impedance (in complex form) expressed as a function of jw. This definition is equivalent to

$$S = -\frac{1}{C_n} - jwR_n \tag{21}$$

and it is therefore seen that constancy of C_n is represented by closeness to a vertical locus, while a small excursion from the negative real axis demonstrates a small loss angle. The locus for $C_n \doteq 0.07$ uf is shown by Fig. 16.

The stability of a negative capacitance in parallel with a positive capacitance (with controlled losses) was examined using the circuit of Fig. 17. The procedure employed was to set the capacitance C at a known value and increase R until oscillations were noted on the cathode-ray oscilloscope. R was then further increased and the type of resulting

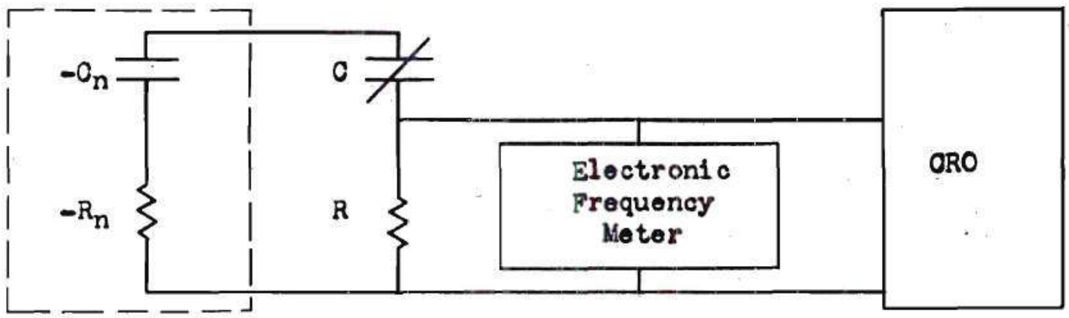


Locus of the Complex Quantity S

Figure 16

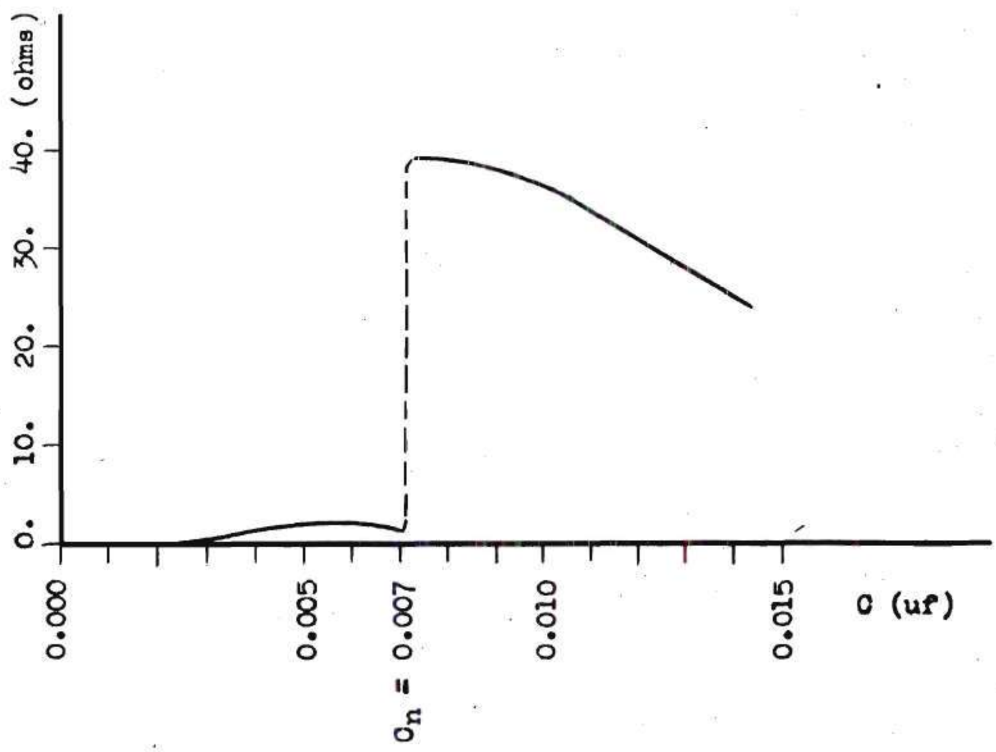
oscillations noted. The frequency of oscillation was also measured with an electronic frequency meter, when the voltage across R was large enough to operate the meter and the frequency was within the 50-kc range of the meter. Outside of this range the frequency was estimated from the oscilloscope trace. No general rules were observed concerning transition from one mode of oscillation to another. The oscillations demonstrated changed character at times gradually and at other times with violent and erratic changes in waveshape. Within a single mode, however, the frequency of oscillation usually was a decreasing function of R and the peak voltage an increasing function of R.

Fig. 18 shows the variation of the critical resistance R_{cr} as a function of C. It illustrates the fact that stability is a practical impossibility for $C < C_n$, but shows that for design purposes it is possible to closely approximate the borderline condition of $C = C_n$ if C_n is not subject to drift. In this case Fig. 18 demonstrates that a sufficient condition for stability is that $R < R_{cr} \doteq R_n$, as would be predicted by the theoretical stability criteria discussed in Chapter II.



Circuit for Investigation of Stability

Figure 17



Variation of the Critical Resistance, R_{cr} With C

Figure 18

CHAPTER IV

COMPENSATION OF AN AMPLIFIER

Statement of the Problem.--Compensation for the shunt capacity in resistance-coupled amplifier circuits by means of negative capacitance has been mentioned in the literature(10). Capacitance neutralization by means of cross-connection of capacitors has long been practiced in radio-frequency amplifiers. The benefits of such neutralization in push-pull transformer-coupled audio amplifiers have been investigated(30) and found to be of little value because of their insignificance in relation to other larger effects, such as multiple transformer resonances. However, to the writer's knowledge, no complete theoretical investigation has been made into the possibility of capacitance cancellation by means of succeeding stages of an amplifier in the case of wide-band, video-type amplifiers.[5] Such compensation might provide better performance or perhaps better economy by improving frequency and phase response without additional tubes and without the necessity for peaking inductances. Actually, compensation with a pure negative capacitance would imply much improved phase and frequency response, without sacrificing one to obtain the other. In fact increases of five or ten to one in band width have been reported by Brunetti(10). On the other hand, the amplifier supplying negative capacitance must be flat beyond the frequency range

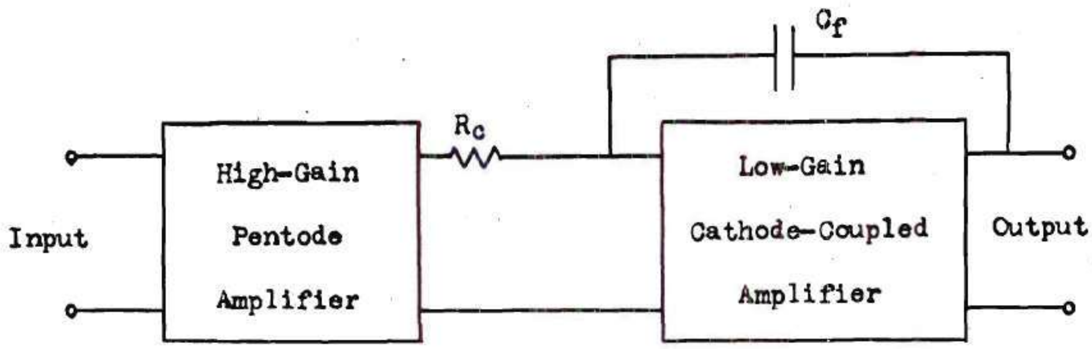
[5]Instances of the use of such compensation have been reported in the literature(31-33).

of the compensated amplifier, and must have a small output impedance. This obviously involves poor economy (although such compensation might be useful in laboratory instrumentation). Therefore an investigation was made to determine the value of capacitance cancellation, using succeeding stages to provide negative capacitance.

Amplifier Requirements for the Compensating Stage.--Since feedback over two stages in the frequency range from 100 kc to 4 mc is difficult to control accurately, it was deemed more practical to apply the capacitive feedback across a single stage. This necessitated the use of a stage which has no polarity reversal.

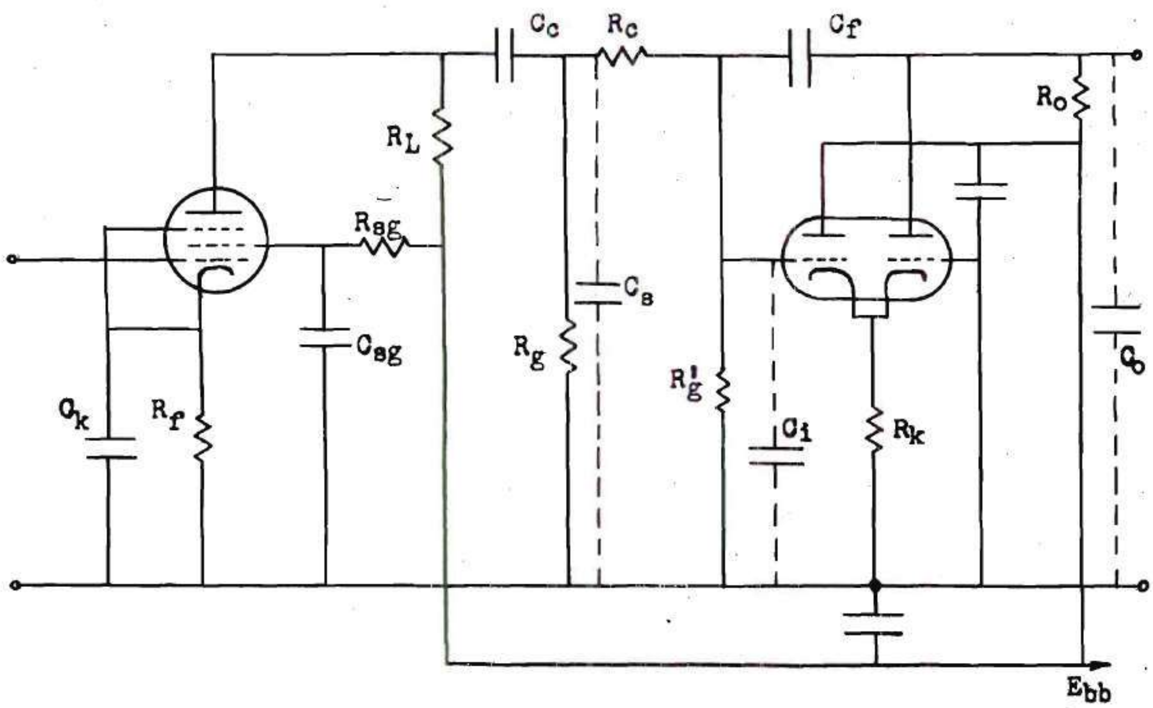
The Cathode-Coupled Amplifier.--The cathode-coupled amplifier, mentioned above in Chapter III, has no polarity reversal from input to output. Its excellent phase characteristics down to zero frequency and at the higher video frequencies make it a suitable choice for the compensating stage. Although this amplifier uses two triodes, for practical purposes it has the characteristics of a single-stage amplifier, since its two triodes are conveniently enclosed in one envelope and since it requires so few elements besides the vacuum tube.

Investigation of the Circuit.--The method used in this investigation was to design a high-gain resistance-coupled amplifier using a sharp cutoff pentode in a conventional circuit. This amplifier was then connected to precede the cathode-coupled stage, and the feedback capacitor C_f was connected directly across the cathode-coupled stage as shown in Fig. 19. The resistance R_c may be thought of as approximately cancelling the negative



Block Diagram of Compensated Amplifier

Figure 19



General Circuit of Compensated Amplifier

Figure 20

resistive component of the negative capacitance-resistance combination caused by the output impedance of the amplifier. This is not strictly so, but may be taken as a sort of first approximation.

The general circuit including all of the important parasitic capacities is shown in Fig. 20. It has been determined that, of the stray capacitances present in the cathode-coupled circuit, only C_i and C_o are of importance in the frequency range of present concern (video frequencies)(34). For the values of C_o encountered experimentally (in the range 1-4 uuf), and considering the magnitude of the amplifier output resistance R_a (less than 10,000 ohms), it was seen that C_o did not play a primary role in the compensation of the amplifier at frequencies near the cutoff frequency of the pentode amplifier. Then the problem was to find optimum values of C_f, A_m, R_c, and R_a to obtain better high-frequency response and increased gain. The derivation of optimum values for these elements is shown in the appendix, as is their detailed application to the amplifier chosen. The practical results of this derivation are as follows:

$$A_m = KR_a \text{ (a large value is desirable)}^{6} \tag{22}$$

$$C_f = \left(\frac{C_s + C_i}{A_m - 1}\right)\left(\frac{R_e}{R_e - R_a}\right) \tag{23}$$

$$R_c = R_a \frac{C_f(C_s + C_i)}{{C_s}^2} \tag{24}$$

---

[6] The value of A_m must not indicate an R_a large enough to become comparable with $1/wC_o$ in the frequency range where compensation is desired.

where A_m is the mid-frequency amplification of the compensating stage, and K is an amplifier transconductance defined by equation 54, appendix.

The physical circuit and its actual element values used in the investigation is shown in Fig. 21. The optimum values for this circuit were calculated (see appendix) taking into account the experimentally determined values for C_s and C_i. The results are as follows.

$A_m \doteq 8.$ (selected)

$A_m = 8.35$ (measured)

$R_a = 7800$ ohm (calculated)

$R_a = 8800$ ohm (measured)

$C_i \doteq 11$ uuf (calculated)

$C_s + C_i = 43.8$ uuf (measured, includes 15 uuf of voltmeter)

$C_s = 12.8$ uuf (measured)

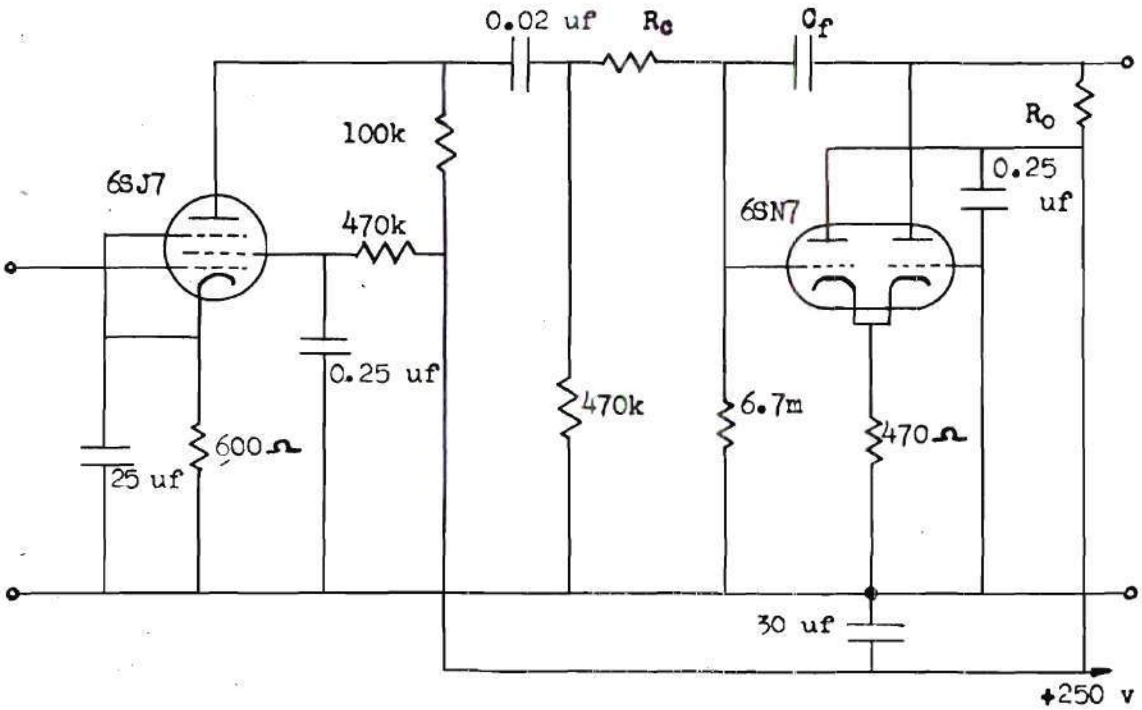
$C_i = 43.8 - 12.8 = 31.$ uuf (including voltmeter)

These results yield optimum values of

$C_f = 6.5$ uuf,

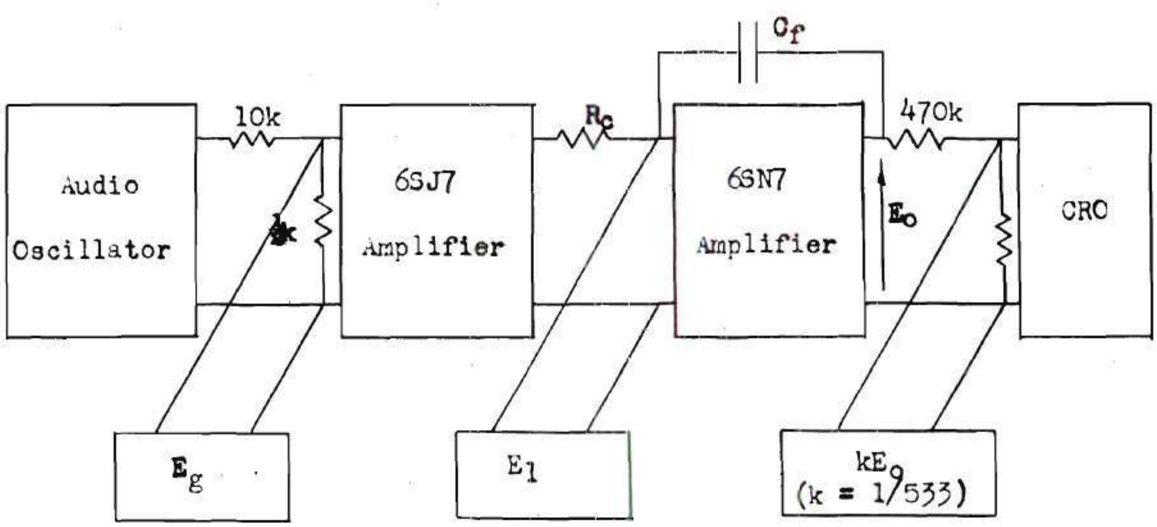
$R_c = 15{,}000$ ohm.

The techniques of measurement used in testing the complete amplifier circuit are illustrated by Fig. 22. The voltmeters used have input capacities of 15 uuf, and their effect could therefore be neglected in measuring E_g and kE_o. However, the impedance level of E_1 is quite high, so that it was necessary to leave the voltmeter E_1 permanently in the circuit, and to include its capacitance as a part of C_i, above. The voltage E_1 was maintained near one volt for all measurements, to avoid over-



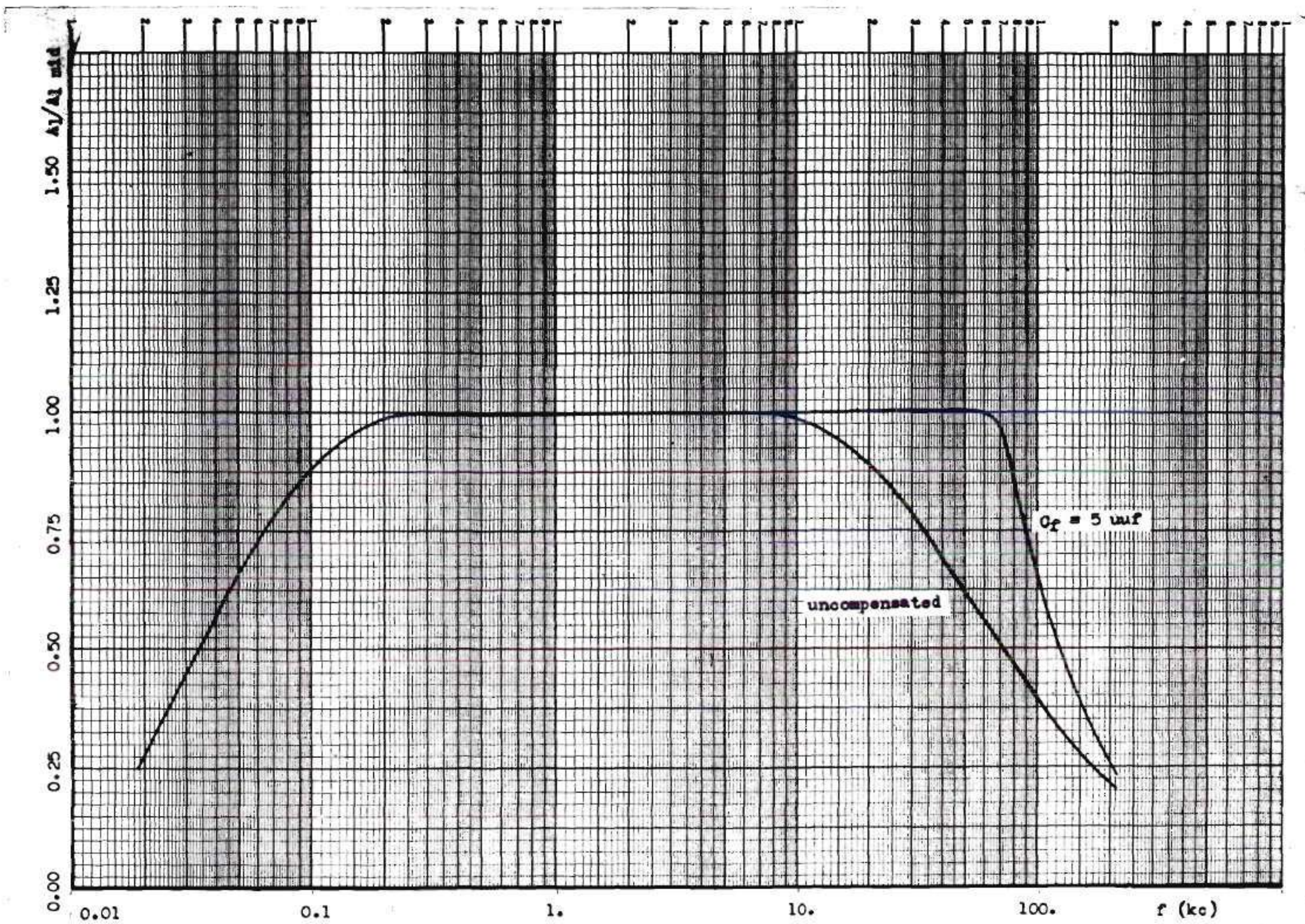
Compensated Amplifier Circuit With Element Values

Figure 21



Measurement of Frequency Response

Figure 22



Amplifier Frequency Response With A_m = 5.2 and R_c = 0

Figure 23

driving the cathode-coupled 6SN7 amplifier. In appropriate instances, of course, R_c and C_f were not employed. In determining C_s it was necessary to disconnect the 6SN7 amplifier from the preceding stage. The cathode-ray oscilloscope was of great utility in rapid estimates of frequency response, detection of instability, and immediate indication of any appreciable nonlinearity in the output waveform. The oscillator employed was of the resistance-capacitance type and had a frequency range of 20-200,000 cps. The output pad was employed to isolate the input capacitance of the vacuum tube voltmeter and oscilloscope from the output impedance of the amplifier.

Fig. 24 shows graphically the results of compensation using R_c = 15,000 ohm and different values of C_f. The voltage amplification of the two stages in cascade is approximately 800. The gain of the cathode-coupled amplifier is substantially constant up to the highest frequency of Fig. 24. At 200 kc the amplification is about one per cent below the mid-frequency value. Accordingly, the figure may be considered as the normalized voltage amplification of either the pentode stage or the complete amplifier for the frequency range shown. It is demonstrated in the appendix that the exact second-stage amplification is

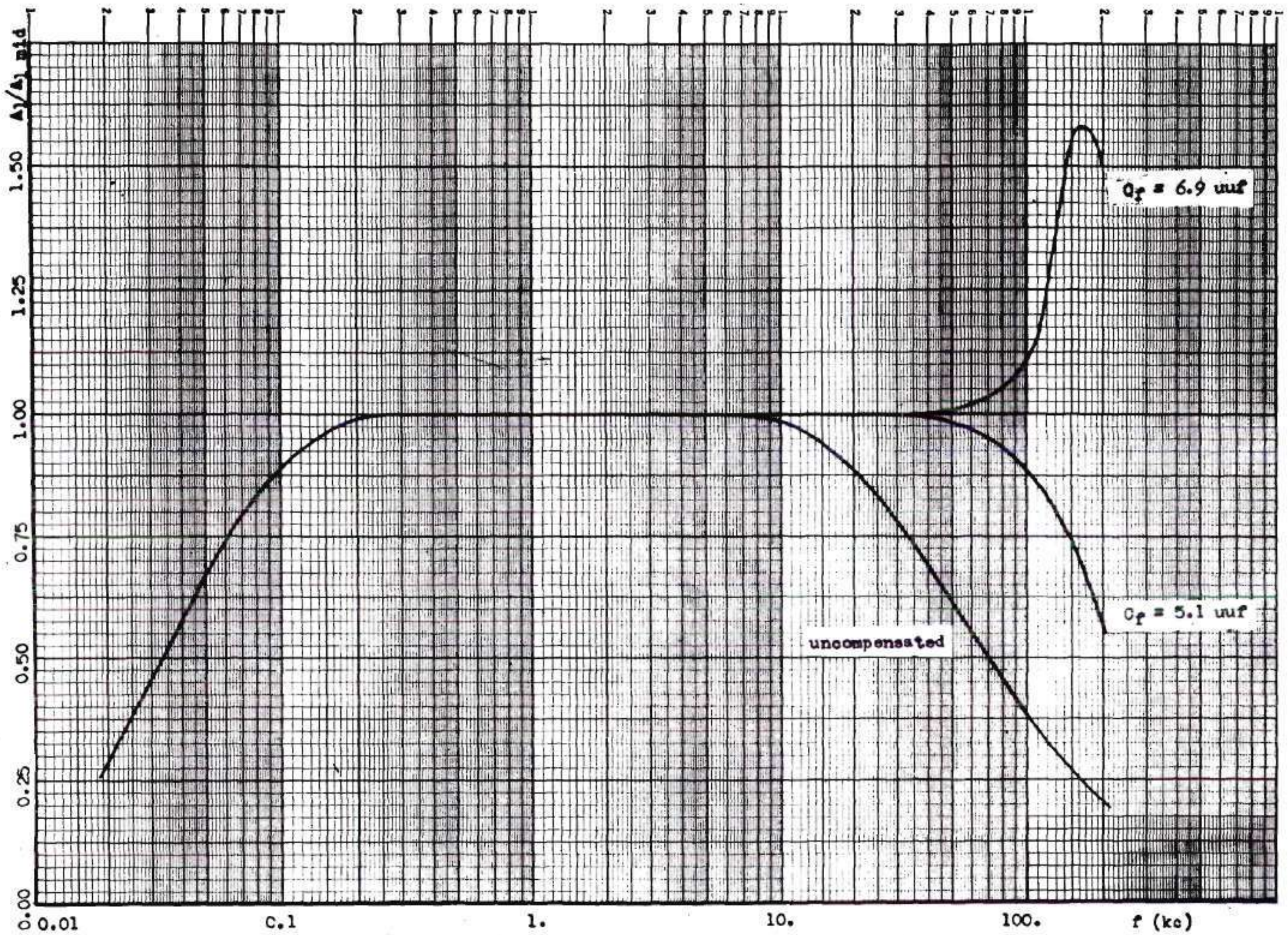
$$\frac{E_o}{E_1} = A_m \frac{1 + jwC_f R_a / A_m}{1 + jwC_f R_a} \tag{25}$$

The response curves of Fig. 25 show the effect of compensation with other than the optimum values of A_m, C_f, and R_c.

The response curve in Fig. 24 representing $C_f = 5.1$ uuf approaches the shape anticipated from the derivation of optimum values for C_f and R_c. The difference between 5.1 uuf and the calculated 6.5 uuf is not excessive considering the possibility of stray capacitances from input to output. The improvement in frequency response is approximately five to one if band-width is measured to the frequency of 90 per cent response. This improvement compares very well with the approximate values of two to one for passive two-terminal compensation and four to one for passive four-terminal compensation (in the most favorable case, where $C_s = C_i$).

Frequency response for larger values of C_f indicates excessive overshoot and a tendency in the direction of oscillation. For values of C_f larger than $C_f = 9$ uuf (with $A_m = 8.35$) oscillation was noted. This indicates a margin for C_f of almost 60 per cent between optimum conditions and oscillatory conditions. With consideration of stray capacities and shielding in the amplifier design this margin was found to be entirely adequate to insure stability with varying line voltages and tube replacements.

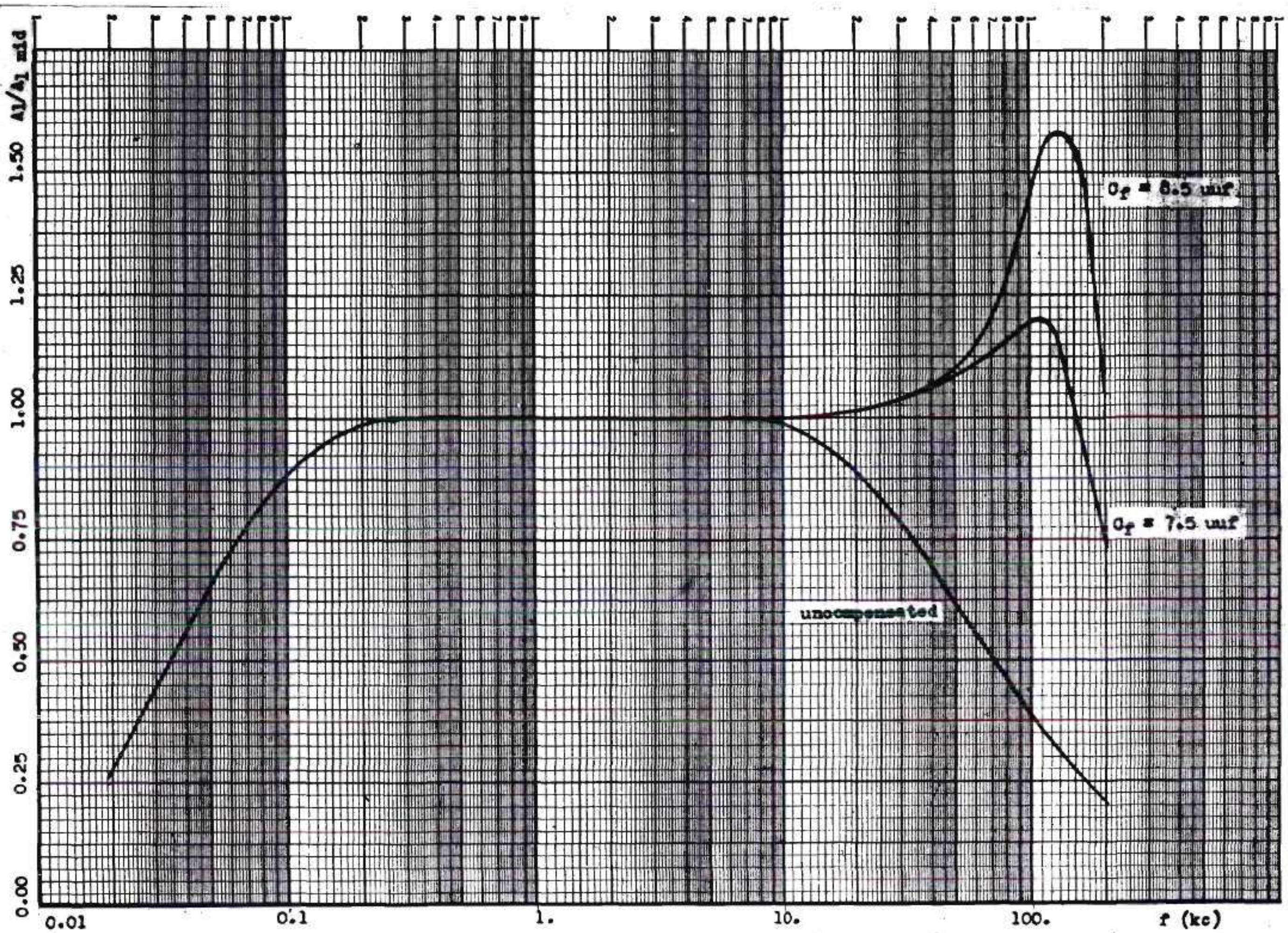
<u>Relative Economy</u>.--The relative economy of this method of compensation can be compared with the use of two pentode amplifiers in cascade. If the pentodes are assumed to have the same characteristics as the 6SJ7 used in the first stage of this amplifier and the same value of $C_s + C_i \doteq 44$ uuf, then each stage must have an equivalent load resistance of approximately 30,000 ohms to obtain a total amplification of 800. Then the total response will be 50 per cent of its mid-frequency value at $f = 120$ kc. At 120 kc the response for the amplifier above



Amplifier Frequency Response With A_m = 8.35 and R_c = 15,000 ohms

Figure 24

is 85 per cent of its mid-frequency value. If both stages of the pentode amplifier employ shunt peaking with the parameter N(35) chosen for greatest similarity of response curve to Fig. 24 with $C_f = 5.1$ uuf ($N = 0.44$), the total amplification is about 90 per cent of its mid-frequency value at $f = 120$ kc. It is therefore evident that the results obtained here are little different from those obtained with shunt peaking, so far as high-frequency response is concerned. However, no inductances have been employed, and the second stage is considerably simpler than a pentode stage, especially since the cathode-coupled amplifier introduces no low-frequency phase-shift due to cathode or screen-grid impedance.



Amplifier Frequency Response With A_m = 6.9 and R_o = 6800 ohms

Figure 25

CHAPTER V

THE VERMAN CIRCUIT FOR A VARIABLE REAL IMPEDANCE

<u>The Basic Circuit</u>.--In Chapter II the basic Verman circuit(Fig. 1) was discussed with particular application to the production of negative capacitance and negative inductance. It was also noted, however, that reinsertion of negative capacitance into the arms of the T-section can produce impedances that are neigher positive nor negative. Such a circuit was investigated in the experimental work for this thesis. The circuit chosen for study is formed by setting $m = 1/jwC$, the impedance of an ordinary capacitance, and $Z = R$, a constant positive resistance. The shunt arm of the T-section is then $-m = 1/-jwC$. A negative capacitance is therefore required. If the question of instability is neglected, substitution into equation 2 yields

$$Z_t = 1/w^2C^2R \tag{26}$$

a real positive impedance which varies inversely as the square of frequency. Ordinarily, an impedance with a real part which changes with frequency must also have an imaginary part which is not identically zero.

A purely resistive element whose resistance varies with frequency could be quite useful in circuit design for filters, equalizers, and other applications since it introduces, of itself, no phase-shift. However, this element as considered above is inherently unstable unless the impedance connected to Z_t has no resistive part. Alternatively Z_t must be driven by a voltage generator with no series resistance in its equivalent circuit.[7] This restriction removes the circuit as shown from the realm of practical utility, but also indicates the type of modification necessary to obtain a practical result.

Approximation to the Desired Impedance.--The modified circuit which permits stability with a non-zero series resistance is shown in Fig. 26. The negative capacitance must possess a series negative resistance $-R_n$, and this fact is taken to advantage to insure stability. The capacitances must be adjusted very precisely to the relation

$$C = C_n \tag{27}$$

This adjustment is again rather critical, since cancellation of the reactive components of Z_t is based on this relation. It is now evident that if R is much larger than R_n, stability will depend primarily upon the value of Z_e (assumed to be resistive only). In fact the circuit for R large is essentially the circuit investigated for stability in Chapter III, above. This investigation demonstrated that the circuit of Fig. 26 will be stable when the capacitances are properly adjusted if

$$Z_e < R_n \tag{28}$$

The change from the ideal circuit to the circuit of Fig. 26 affects considerably the expression for Z_t. It is shown in the appendix that

[7] This statement presupposes the use of a shunt-type negative capacitance in the circuit.

the relation now is

$$Z_t = \frac{1}{w^2c^2R}\left(\frac{1 + j2wR_nC - w^2c^2R_nR}{1 - R_n/R}\right) \tag{29}$$

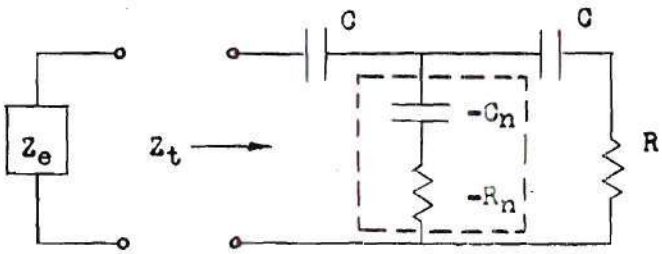
This impedance differs from that of equation 26 in two essential ways. First, it has a non-zero imaginary part; and, second, its real part does not remain positive for all frequencies. These changes represent the concessions necessary to insure stability.

It is worthwhile to examine equation 29 more fully. In a practical application, such as a filter, it would probably be desirable to control both the imaginary part of Z_t and the frequency where the real part of Z_t changes sign. It might also be desirable to specify R_n arbitrarily to insure stability with some particular driving source. Suppose, for example, it is desired that the network remain stable when $Z_e = K_o$. A value of $R_n = K \geqslant K_o$ ohms might then be selected to insure stability. The most important frequency made evident by equation 29 is the zero of the real part of Z_t, which occurs at

$$w^2c^2R_nR = 1 \tag{30}$$

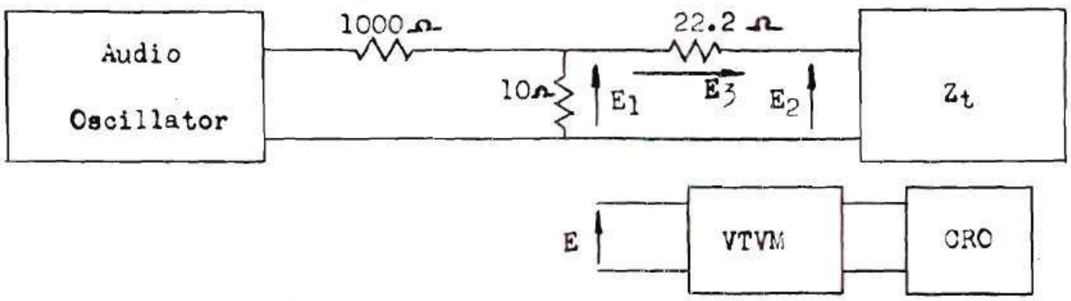
If this zero is desired to fall at some particular frequency, say w_o, then a specification of the magnitude of the imaginary part may be based on the value of $2w_oR_nC$ compared with unity. If for instance it is specified that

$$2w_oR_nC = \alpha < 1 \tag{31}$$



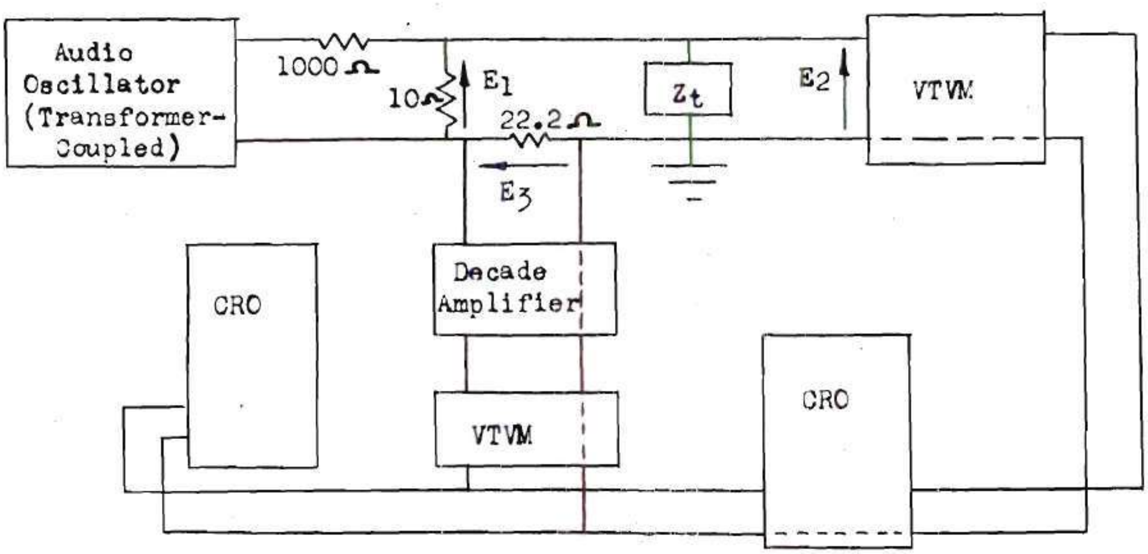
Practical Form of the Verman Circuit

Figure 26



First Circuit for Measuring Z_t

Figure 27



Second Circuit for Measuring Z_t

Figure 28

then, since $R_n \doteq K$ and w_o are constants already determined, C is now fixed. These constraints then leave only R to be varied for the satisfaction of equation 30. Specification of any additional characteristics, such as the magnitude of Z_t at some frequency other than w_o, leads either to an inconsistency or a redundant condition on Z_t.

The circuit of Fig. 26 was investigated experimentally using as the shunt branch of the T-section the negative capacitance discussed in Chapter III. The circuit for which $C_n \doteq 0.007$ uf and $R_n \doteq 35$ ohms was employed, and a value of 500 ohms was chosen for R. Precision decade capacitors shunted with vernier capacitors were employed in the series arms of the T-section. The capacitance C was set at 0.007106 uf.

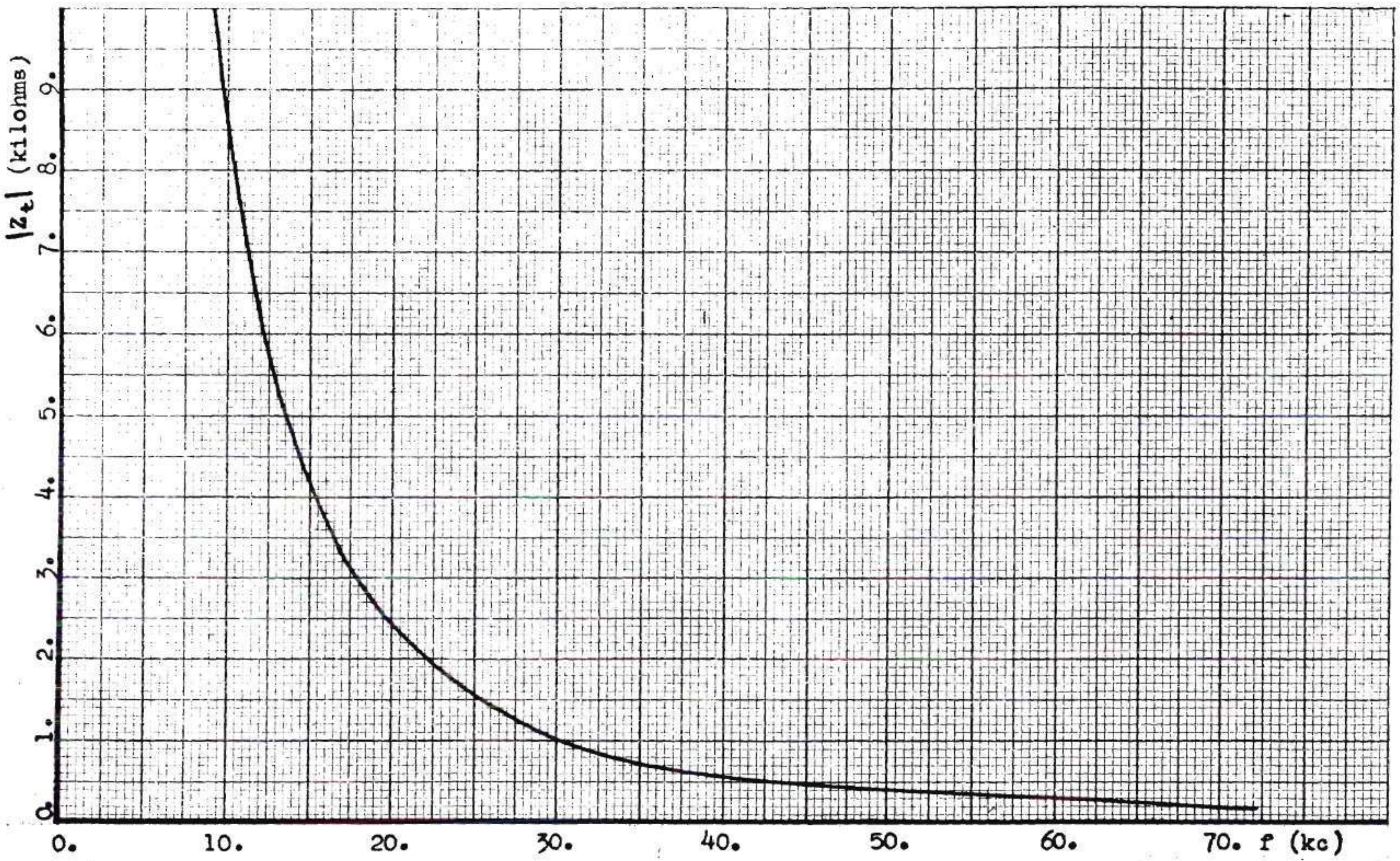
If R_n were a positive constant, equation 29 would indicate at any one frequency an inductive reactance in series with a resistance. However, in the case where $-R_n$ is variable, as shown by Fig. 15, the resistive part of Z_t may be effectively in series with either an inductive or capacitive reactance. Furthermore, slight variation in C_n can easily cause enough variation of this reactive component to change its algebraic sign. Therefore the circuit was expected to yield the proper variation of the real part of Z_t, but the imaginary part was not expected to follow any easily predictable pattern. It nevertheless seems reasonable to suppose that the imaginary component of Z_t would be fairly small over some range of frequencies.

Measurement Techniques.--The discussion above concerning the number of independent constraints in the use of the general impedance Z_t has particular application to the problem of accurately measuring Z_t. The measuring circuit connected to Z_t must not have an internal impedance (assumed resistive) of greater than about 30 to 35 ohms. The application of equation 29 yeilds for Z_t at frequencies well below 140 kc (the approximate frequency of the zero of the real component) the approximate expression

$$Z_t = \frac{1005}{f^2} \text{ (kilohms)} \tag{32}$$

where f is measured in kilocycles per second. At a frequency of 10 kc this relation shows Z_t to be about 10,000 ohms. A technique for measuring Z_t was needed which could measure impedances of such magnitude and still present to the active network a resistive impedance of no more than 35 ohms.

The method employed was the standard three-voltmeter method, since this method allows control of the impedance of the measuring circuit. The circuits employed are shown by Fig. 27 and Fig. 28. In using either of these circuits a resistance appears between the ground terminals of two of the instruments. This difficulty was accepted since the resistance in either case was small, and since no other method of measurement was feasible. Theoretically the three-voltmeter method yields both magnitude and angle of the measured impedance. In this case voltages E_1 and E_2 were so nearly equal (E_3 was ordinarily of the same order of magnitude as the observational error involved in reading E_1 and E_2) that no phase measurement could be made. However by using the circuit of Fig. 28 a phase measurement was possible for frequencies above about 15 kc using the Lissajous pattern on the cathode-ray oscilloscope screen.



Variation of $|Z_t|$ With Frequency

Figure 29

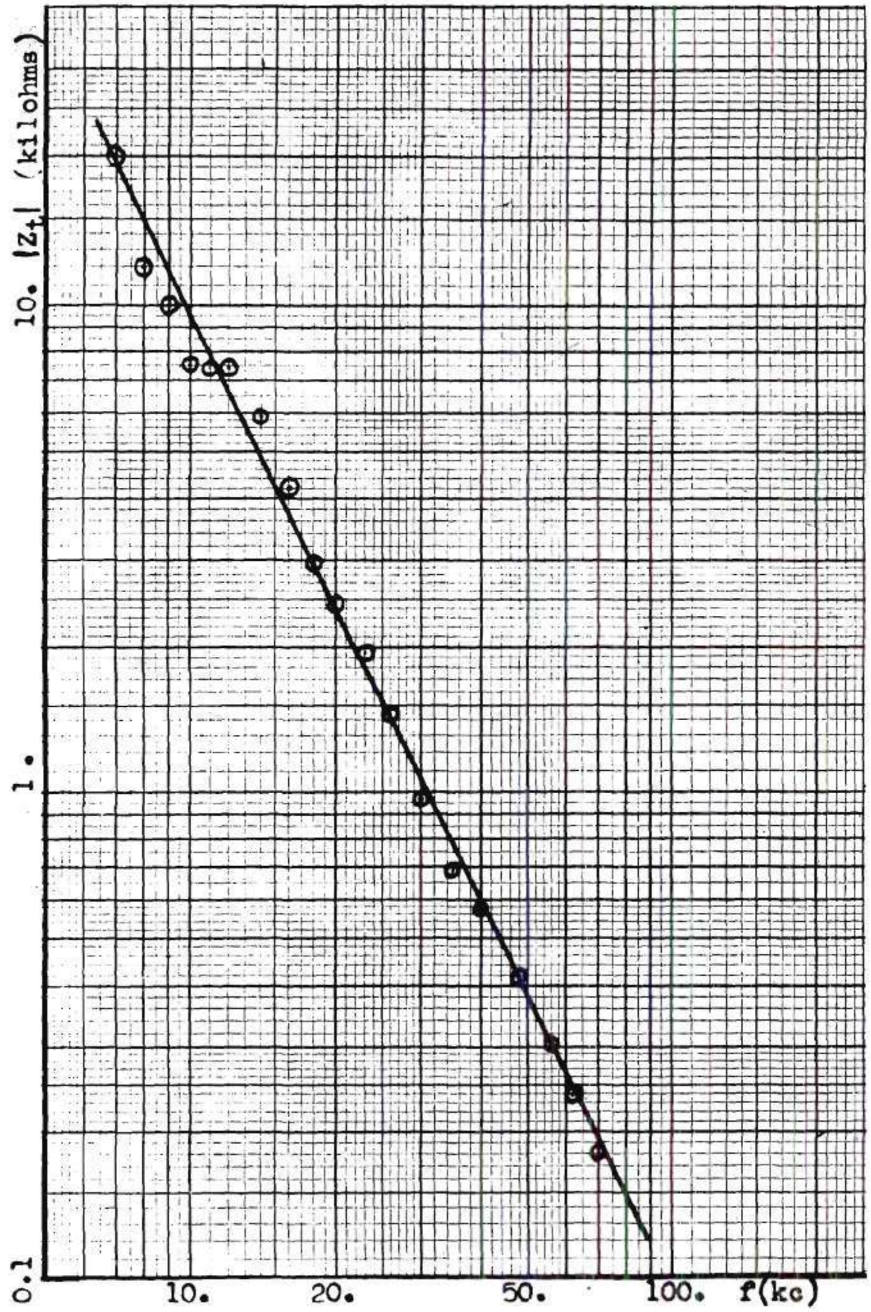
The phase difference measured by this pattern is the angle between E_2 and E_3 if there is no phase-shift in the voltmeter and oscilloscope amplifiers. At frequencies above about 30 kc phase-shift in the amplifiers became too large and unpredictable (for various gain settings) to allow satisfactory phase measurements. It was deemed impractical to attempt any measurements at frequencies higher than 70 kc, since the high-frequency limit of most of the equipment used was in the range 70 to 150 kc.

The effects of hum, noise and power-line transients were so large in comparison with the voltage E_3 for frequencies below about 14 kc that the measurements at these frequencies must be considered quite approximate.

Experimental Results.--The variation of $|Z_t|$ with frequency, determined from the circuit of Fig. 27, is shown in Fig. 29, in which the abcissa and ordinate scales are linear. A somewhat more useful method of demonstrating the variation of $|Z_t|$ with frequency is shown in Fig. 30. In this figure both abcissa and ordinate scales are logarithmic and points representing the experimental data have been explicitly plotted to show the degree of scattering from the ideal variation. A line with a slope of two decades per decade has been drawn through the estimated centroid of the points. The equation of this line is

$$|Z_t| = \frac{913}{f^2} \text{ (kilohms)} \tag{33}$$

with f again measured in kilocycles per second. The close correlation between the points and this line, and the reasonably close agreement



Variation of $|Z_t|$ (Measured Using Circuit of Figure 27) With Frequency

Figure 30

between equations 32 and 33 indicate the validity of equation 29 up to frequencies of about $\frac{1}{2}w_o$. If the variation expressed by equation 33 is extrapolated to a frequency of 140 kc, a value of $|Z_t| = 46.5$ ohms is obtained. This result agrees with the predicted crossover frequency of 140 kc since 46.5 ohms is within the range of values associated with R_n above 10 kc. It should probably be emphasized that this discussion has been based only on the measured magnitude of $|Z_t|$ and not its associated angle.

The variation of Z_t versus frequency obtained using the circuit of Fig. 28 is shown in Fig. 31. Again the points scatter closely about a line with a slope of two decades per decade. The equation of this line is

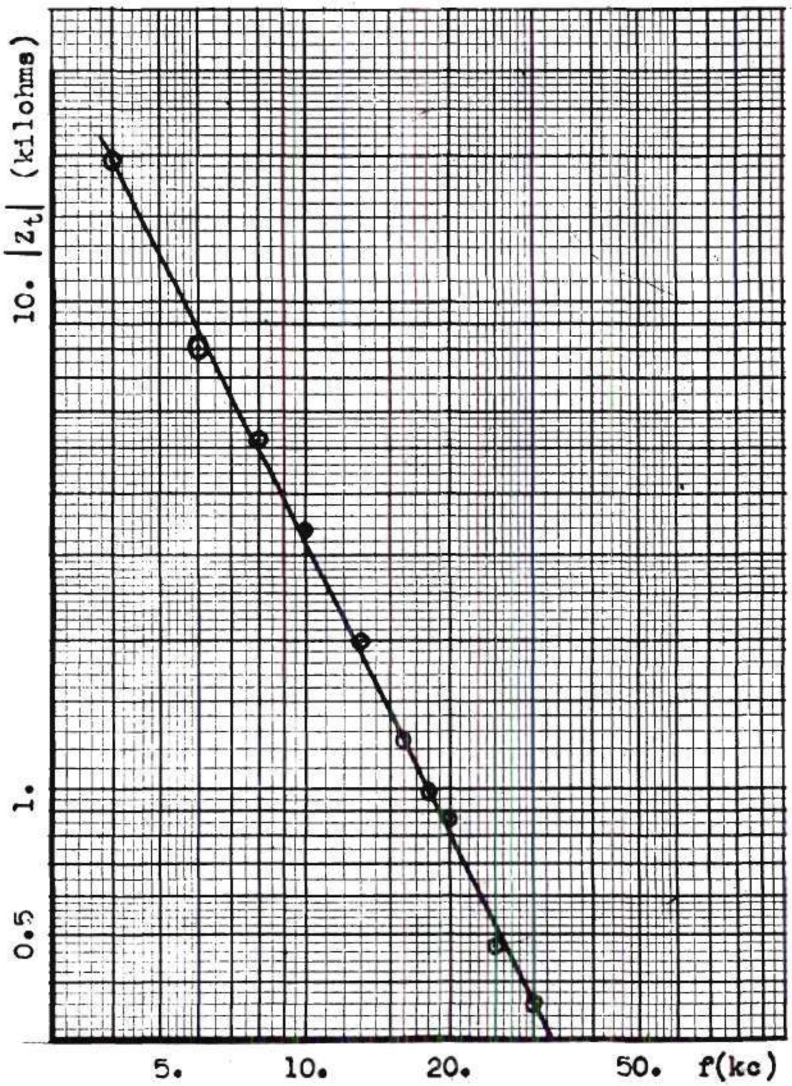
$$|Z_t| = \frac{320}{f^2} \tag{34}$$

a relation which agrees very poorly with equation 32. The difference between this result and equation 33 is believed to be due to the fact that the voltmeters employed in the circuit of Fig. 27 are of the peak-reading type, while those employed in the circuit of Fig. 28 are of the average-reading type.

The results of the measurement of the angle θ, where

$$Z_t = |Z_t| \underline{/\theta} \tag{35}$$

are shown in Fig. 32. Because of slight nonlinearities in the circuit (primarily in the negative capacitance circuit), the Lissajous pattern used to measure θ was not perfectly elliptical. For the fairly small

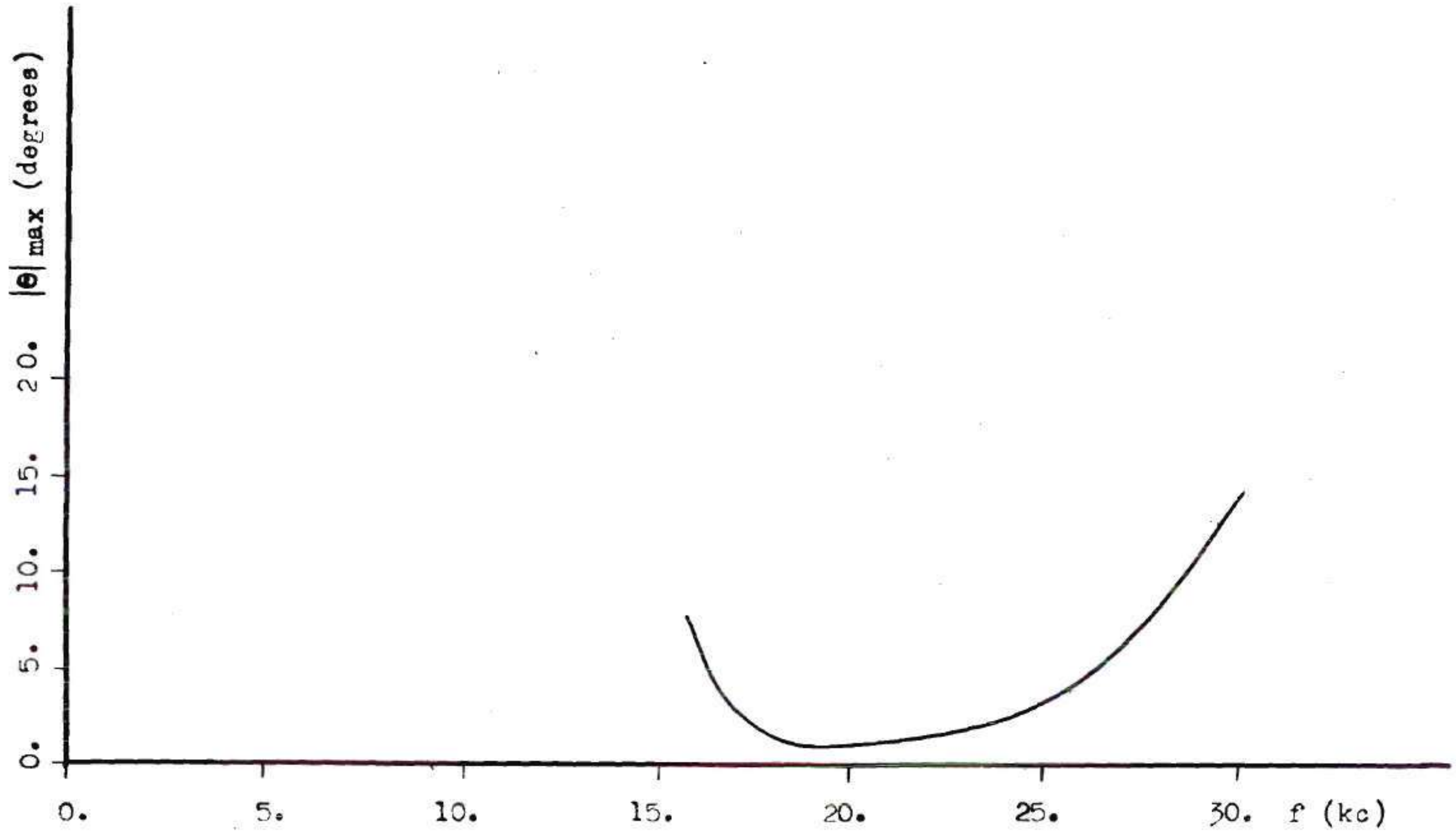


Variation of $|Z_t|$ (Measured Using Circuit of Figure 28) With Frequency

Figure 31

values of sin θ encountered, the loop was almost closed and the amplitude distortion often caused the sides of the loop to interlace. On this interlaced pattern it was difficult to determine sin θ very accurately, and an indisputable determination of the sense of θ was a practical impossibility. For these reasons the maximum value of θ has been plotted in Fig. 32. In the limited range of frequencies shown in Fig. 32 the phase-angle measurement tends somewhat to verify the theoretical predictions, but the data is actually too meager to support any general statement.

The salient features of the behavior of the active network as expressed by equation 29 have been partially verified by several results discussed in this section. However, it is quite evident that the most satisfactory of these results is the close agreement between equations 32 and 33 and the impressive correlation between the plotted points of Fig. 30 and the line representing equation 33.



Variation of Maximum Measured Angle With Frequency

Figure 32

CONCLUSIONS

A practical shunt-type negative capacitance was constructed to approximate closely the behavior of a pure negative capacitance over a frequency range of about two decades. The most important parasitic element associated with this negative capacitance was a series negative resistance. This negative resistance varied somewhat with frequency, but was substantially constant in the mid-frequency range of the amplifier employed in producing the negative capacitance. When the negative capacitance was connected to the series combination of a positive (passive) resistance and capacitance, the condition for which the positive and negative capacitances were equal in magnitude could be closely approximated with stability if the positive resistance was less in magnitude than the series negative resistance associated with the negative capacitance.

It was found possible to use a shunt-type negative capacitance circuit to compensate for the shunt capacitance of a pentode amplifier over a frequency range of about five to one. At the same time additional amplification was obtained from the compensating amplifier. The overall results of such compensation were little different from those obtained with two pentode stages employing shunt peaking so far as high-frequency response is concerned. However, no inductances were required, and the very simple compensating stage introduced no low-frequency phase-shift due to screen-grid or cathode impedance.

A proposed circuit for realizing a resistive impedance inversely proportional to the square of the frequency was found to be unstable in its ideal form, but stable in its practical form when certain restrictions were placed on the driving source for the impedance. In the practical form of the circuit, the impedance had a small but non-zero reactive component; and the resistive component, even in theory, could approximate the desired inverse square relation only within a limited frequency range.

RECOMMENDATIONS

It is recommended that further study be made of the possibilities of application of negative impedances in problems of network synthesis and in practical circuit design. Furthermore, an investigation of the potential value of small battery-powered, self-enclosed negative impedances using low-drain transistor amplifiers would probably be useful.

APPENDIX I

DERIVATIONS

Derivation of Negative Impedance Using Verman's Circuit (Fig. 1).--

$$Z_t = m + \frac{(-m)(m + Z)}{-m + m + Z}$$

$$Z_t = m - \frac{m^2}{Z} - \frac{mZ}{Z} = \frac{-m^2}{Z} \tag{2}$$

Derivation of Negative Impedance Using Series-Type Circuit (Fig. 3).--

Assume a current source, I. Then

$$\begin{aligned} e &= IR_2 + Ae_1 + IR_n + IZ_n \\ e_1 &= -I(R_n + Z_n) \end{aligned} \tag{36}$$

or

$$e = I\left[R_2 + R_n - AR_n + Z_n - AZ_n\right] \tag{37}$$

Let

$$R_2 = R_n(A-1) \tag{38}$$

Then

$$\frac{e}{I} = -Z_n(A - 1)$$

and therefore

$$Z_t = -Z_n(A - 1) \tag{39}$$

Derivation of Negative Impedance Using Shunt-Type Circuit (Fig. 4).--
The simultaneous voltage equations are

$$\begin{aligned} e &= Ae_1 + IR_3 + IZ_n + IR_2 \\ e_1 &= -IR_3 + e \end{aligned} \tag{40}$$

or

$$e = -IR_3(A - 1) + IR_2 + IZ_n + Ae$$

$$\frac{e}{I}(1 - A) = R_2 - R_3(A - 1) + Z_n \tag{41}$$

Let

$$R_2 = R_3(A - 1) \tag{42}$$

Then

$$Z_t = \frac{e}{I} = \frac{Z_n}{1 - A} = \frac{-Z_n}{A - 1} \tag{43}$$

<u>Derivation of Voltage Amplification and Output Impedance of the Cathode-Coupled Amplifier (Fig. 12)</u>.--The equivalent circuit is drawn in Fig. 33. The method to be followed is the determination of the Norton Equivalent Generator of Fig. 34. Resistance R_2 may be removed from consideration since it appears in shunt with the output only, and loop current i_2 may be closed through the voltage source, E.

In the equivalent circuit of Fig. 33, the voltages e_{g1} and e_{g2} are given by

$$e_{g1} = E_1 + V_{ac} = E_1 - V_{ca} \tag{44}$$

$$e_{g2} = V_{ac} = -V_{ca} \tag{45}$$

where E_1 is the input voltage. If the generators driving R_k are replaced by Norton Equivalent Circuits, Fig. 35 results. Now the required parameters are

$$I_g = i_2 \Big|_{E = 0} \tag{46}$$

and

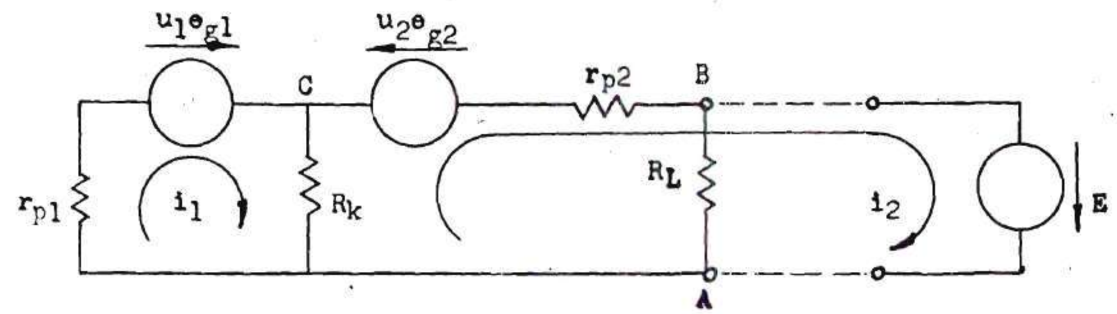
$$R_p = \frac{E}{i_2} \Big|_{E_1 = 0} \tag{47}$$

The node equation is

$$V_{ca} = \frac{g_{m1}e_{g1} + g_{m2}e_{g2} - E/r_{p2}}{1/r_{p1} + 1/R_k + 1/r_{p2}} \tag{48}$$

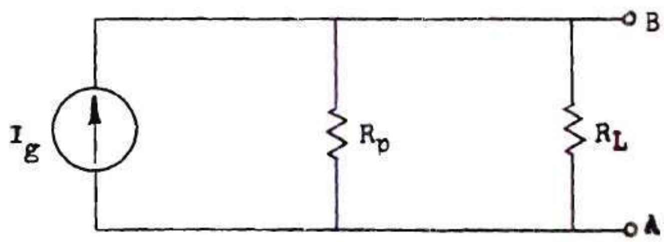
Let

$$1/M = 1/r_{p1} + 1/R_k + 1/r_{p2} \tag{49}$$



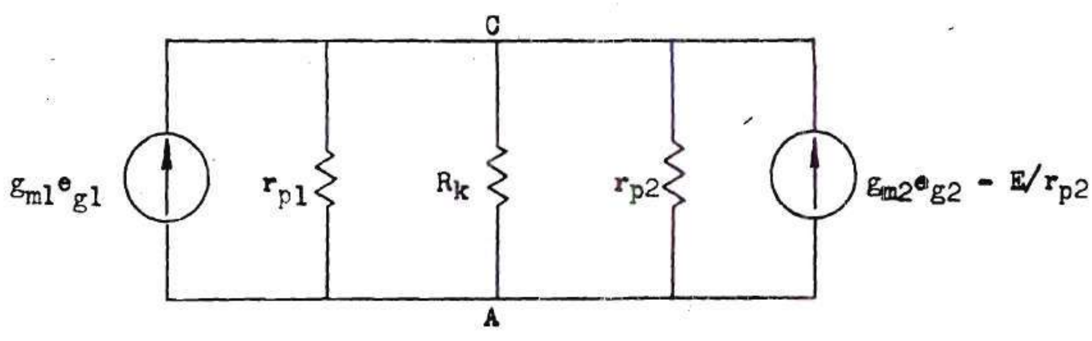
Equivalent Circuit of Cathode-Coupled Amplifier

Figure 33



Norton Equivalent Circuit of Cathode-Coupled Amplifier

Figure 34



Equivalent Circuit for C alculating V_{ca}

Figure 35

then

$$V_{ca} = Mg_{m1}(E_1 - V_{ca}) + Mg_{m2}(-V_{ca}) - ME/r_{p2}$$

or

$$V_{ca}\left[1 + Mg_{m1} + Mg_{m2}\right] = Mg_{m1}E_1 - ME/r_{p2}$$

$$V_{ca} = \frac{g_{m1}E_1 - E/r_{p2}}{g_{m1} + g_{m2} + 1/M} \tag{50}$$

but

$$i_2 = \frac{V_{ca} + E - u_2 eg_2}{r_{p2}} = \frac{(u_2 + 1)V_{ca} + E}{r_{p2}} \tag{51}$$

and

$$I_g = i_2\Big|_{E=0} = \frac{u_2 + 1}{r_{p2}} V_{ca}\Big|_{E=0} \tag{52}$$

Therefore

$$I_g = \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{g_{m1}E_1}{g_{m1} + g_{m2} + 1/M}\right) = KE_1 \tag{53}$$

where

$$K = \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{g_{m1}}{g_{m1} + g_{m2} + 1/M}\right) \tag{54}$$

is the effective amplifier transconductance.

Considering i_2 again, we have

$$i_2 = E/r_{p2} + \left(\frac{u2 + 1}{r_{p2}}\right)V_{ca} \tag{51}$$

and if R_p is desired, we are interested in

$$i_2' = i_2 \Big|_{E_1 = 0} \tag{55}$$

$$i_2' = E/r_{p2} + \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{-E/r_{p2}}{g_{m1} + g_{m2} + 1/M}\right)$$

or

$$i_2' = \frac{E}{r_{p2}}\left[1 + \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{-1}{g_{m1} + g_{m2} + 1/M}\right)\right] \tag{56}$$

which yields

$$R_p = \frac{E}{i_2'} = \frac{r_{p2}}{1 - \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{1}{g_{m1} + g_{m2} + 1/M}\right)} \tag{57}$$

Another convenient form may be obtained by expanding

$$\frac{i_2' r_{p2}}{E} = 1 - \left(\frac{u_2 + 1}{r_{p2}}\right)\left(\frac{r_{p1} r_{p2} R_k}{u_1 r_{p2} R_k + u_2 r_{p1} R_k + r_{p2} R_k + r_{p1} R_k + r_{p1} r_{p2}}\right)$$

$$= \frac{r_{p2} R_k(u_1 + 1) + r_{p1} R_k(u_2 + 1) + r_{p1} r_{p2} - r_{p1} R_k(u_2 + 1)}{r_{p2} R_k(u_1 + 1) + r_{p1} R_k(u_2 + 1) + r_{p1} r_{p2}}$$

$$= r_{p2} \frac{R_k(u_1 + 1) + r_{p1}}{r_{p2} R_k(u_1 + 1) + r_{p1} R_k(u_2 + 1) + r_{p1} r_{p2}}$$

then

$$R_p = \frac{E}{i_2'} = \frac{r_{p1} r_{p2} + r_{p2} R_k(u_1 + 1) + r_{p1} R_k(u_2 + 1)}{r_{p1} + R_k(u_1 + 1)} \tag{58}$$

If this result is introduced into

$$A_m = KR_a \tag{22}$$

where

$$\frac{1}{R_a} = \frac{1}{R_p} + \frac{1}{R_L} \tag{23}$$

equation 17 will result. Equation 19 results from the introduction of 57 into 23.

Continuing with R_p, we have by division

$$R_p = r_{p2} + \frac{r_{p1}R_k(u_2 + 1)}{r_{p1} + R_k(u_1 + 1)} \tag{59}$$

In the ordinary case where

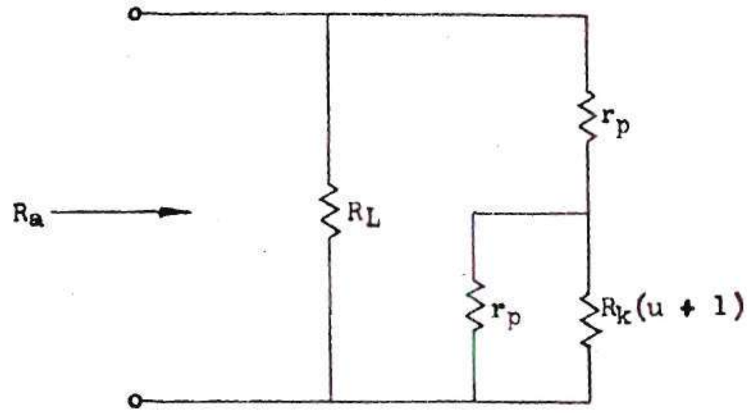
$$u_1 = u_2 = u$$

and

$$r_{p1} = r_{p2} = r_p$$

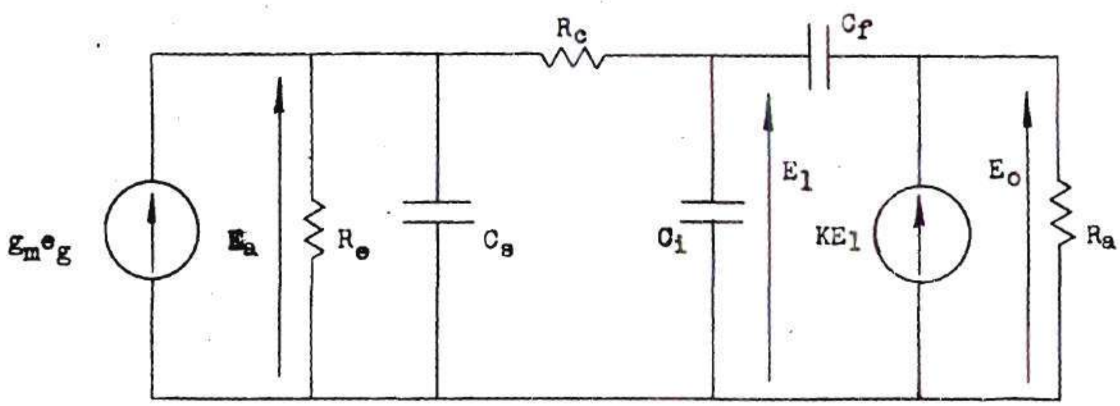
the equivalent circuit of Fig. 36 is seen to hold.

Derivation of Optimum Element Values for Compensated Amplifier.--The equivalent circuit for the compensated amplifier (Fig. 20) is shown in Fig. 37. In this equivalent circuit both the pentode stage and the cathode-coupled stage have been replaced by their Norton Equivalent Generators, and the circuit drawn is valid for small signals. Furthermore it has been assumed that C_o is small enough to be neglected at the



Equivalent Circuit for R_a

Figure 36



Circuit for Analysis of Compensated Amplifier

Figure 37

frequencies considered, and that R_g' is large enough to be omitted for a-c calculations. Parameters K and R_a are defined by equations 54 and 18 respective, and in the pentode stage R_e is the usual output resistance, where

$$\frac{1}{R_e} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g} \tag{60}$$

Writing node equations, we have

$$\begin{aligned} g_m e_g &= E_a Y_a - E_1 G_c \\ 0 &= -E_a G_c + E_1 Y_1 - E_o B_f \\ KE_1 &= \qquad -E_1 B_f + E_o Y_o \end{aligned} \tag{61}$$

where

$$\begin{aligned} Y_a &= 1/R_e + 1/R_c + jwC_s \\ Y_1 &= 1/R_c + jwC_f + jwC_i \\ Y_o &= 1/R_a + jwC_f \\ G_c &= 1/R_c \\ B_f &= jwC_f \end{aligned} \tag{62}$$

Transposing, we have

$$\begin{aligned} g_m e_g &= E_a Y_a - E_1 G_c \\ 0 &= -E_a G_c + E_1 Y_1 - E_o B_f \\ 0 &= \qquad - E_1 Y_m + E_o Y_o \end{aligned} \tag{63}$$

where

$$Y_m = K + jwC_f$$

To solve by determinants we write

$$\Delta_s = \begin{vmatrix} Y_a & -G_c & 0 \\ -G_c & Y_1 & -B_f \\ 0 & -Y_m & Y_o \end{vmatrix} = Y_a \left[Y_1 Y_o - Y_m B_f \right] + G_c \left[-G_c y_o \right] \tag{64}$$

and

$$\Delta_o = \begin{vmatrix} Y_a & -G_c & g_m e_g \\ -G_c & Y_1 & 0 \\ 0 & -Y_m & 0 \end{vmatrix} = g_m e_g G_c Y_m \tag{65}$$

Expanding, we obtain

$$\Delta_o = \frac{g_m r_e e_g}{R_c R_a R_e} \left[A_m + jwC_f R_a \right] \tag{66}$$

where A_m is given by equation 22.

Expansion of equation 64 yields

$$s = \frac{1}{R_e R_a R_c^2} \Bigg[\Bigg[R_c + jw\Big\{ C_s R_e R_c + (R_e + R_c) \left[C_i R_c - C_f \ (A_m - 1) R_c \right] + C_f R_a R_c \Big\} - w^2 \Big\{ C_s R_e R_c \left[C_i R_c + C_f R_a - C_f (A_m - 1) R_c \right] + (R_c + R_e) C_f C_i R_a R_c \Big\} - jw^3 \Big\{ C_s R_e C_f R_a C_i R_c^2 \Big\} \Bigg]\Bigg] \tag{67}$$

Now if we assume (as in the practical case) that

$$R_e >> R_c \tag{68}$$

and simplify equation 68, we obtain

$$\Delta_s = \frac{1}{R_e R_a R_c} \Bigg[\Big[1 + jw \Big\{ \big[C_s + C_i - C_f(A_m - 1) \big] R_e + C_f R_a \Big\}$$

$$-w^2 \Big\{ C_f R_a \big[C_s R_e + C_i R_e \big] + C_s R_e \big[C_i - C_f (A_m - 1) \big] R_c \Big\}$$

$$-jw^3 C_s R_e C_f R_a C_i R_c \Big]\Bigg] \tag{69}$$

Now E_o may be calculated

$$E_o = \frac{\Delta_o}{\Delta_s} = \frac{g_m R_e e_g \big[A_m + jwC_f R_a \big]}{1 + jwT_1 - w^2 T_2 - jw^3 T_3} \tag{70}$$

where

$$T_1 = \big[C_s + C_i - C_f(A_m - 1) \big] R_e + C_f R_a$$

$$T_2 = C_f R_a \big[C_s R_e + C_i R_e \big] + C_s R_e \big[C_i - C_f(A_m - 1) \big] R_c \tag{71}$$

$$T_3 = C_s R_e C_f R_a C_i R_c$$

Equation 70 may be rewritten as

$$\frac{E_o}{e_g} = \frac{g_m R_e A_m \big[1 + jwC_f/K \big]}{1 + jwT_1 - w^2 T_2 - jw^3 T_3} \tag{71}$$

Now, to obtain a flat gain characteristic to as high a frequency as possible (without overshoot), it is necessary to set

$$T_1 = C_f/K \tag{72}$$

$$T_2 = 0 \tag{73}$$

and to minimize T_3. From equation 72 we obtain the optimum value of

$$C_f = \left(\frac{C_s + C_i}{A_m - 1}\right)\left(\frac{R_e}{R_e - R_a}\right) \tag{74}$$

and from equation 73 the optimum value of

$$R_c = R_a \frac{C_f(C_s + C_i)}{C_s^{\,2}} \tag{75}$$

Now, to minimize T_3, we write the approximate relation

$$C_f = (C_s + C_i)/A_m \tag{76}$$

Then

$$T_3 = \frac{(C_s + C_i)^3 \, C_i R_e}{K^2 C_s} \tag{77}$$

a result which is independent of A_m. Therefore a large value of A_m should be employed to give a large gain-bandwidth product. Then, if desired, R_e may be decreased to decrease T_3 and obtain a greater bandwidth.

If E_o/E_1 is of interest, it may be obtained easily as

$$\frac{E_o}{E_1} = \frac{\Delta_o}{\Delta_1}$$

where

$$\Delta_1 = \begin{vmatrix} y_a & g_m e_g & 0 \\ -G_c & 0 & -B_f \\ 0 & 0 & Y_o \end{vmatrix} = G_c Y_o g_m e_g$$

$$= \frac{g_m e_g}{R_c R_a} (1 + jwC_f R_a) \tag{78}$$

Therefore

$$\frac{E_o}{E_1} = \frac{A_m + jwC_f R_a}{1 + jwC_f R_a}$$

or

$$\frac{E_o}{E_1} = A_m \left(\frac{1 + jwC_f R_a / A_m}{1 + jwC_f R_a} \right) \tag{25}$$

Derivation of Input Impedance of the Network of Fig. 26.--The condition of interest is that where

$$C = C_n \tag{27}$$

$$Z_t = \frac{1}{jwC} + \frac{-(R + \frac{1}{jwc})(R_n + \frac{1}{jwc})}{R - R_n}$$

$$= \frac{1}{jwc} + \frac{-R_n R - \frac{R_n}{jwc} - \frac{R}{jwc} + \frac{1}{w^2 c^2}}{R - R_n}$$

$$= \frac{-R_n R - \dfrac{2R_n}{jwc} + \dfrac{1}{w^2c^2}}{R - R_n}$$

$$= \left(\frac{-w^2c^2R_nR + j2wR_nC_n + 1}{R - R_n} \right) \frac{1}{w^2c^2}$$

$$Z_t = \frac{1}{w^2c^2R} \left(\frac{1 + j2wR_nC_n - w^2c^2R_nR}{1 - R_n/R} \right) \tag{29}$$

APPENDIX II

Table 1. Values of Components and Parameters of Cathode-Coupled Amplifier (Fig. 12) Employing 6J6 Type Vacuum Tube

Parameter of Component	Value
R_k	390 ohms
R_L	1800 ohms
R_g	470 kilohms
R_a	1800 ohms
A_m	2.04

Table 2. Frequency Response of Negative Capacitance Produced with Cathode-Coupled Amplifier and Employing $C_f = 0.125$ uf

f (kc)	C_{p1} (uf)	C_{p2} (uf)	R_{p1} (ohms)	R_{p2} (ohms)	$-C_n$ (uf)	$-R_n$ (ohms)
0.5	0.680	0.598	4174.	16,784.	-0.122	-1820.
1.0	0.641	0.596	1600.	4,232.	-0.130	-1690.
4.0	0.566	0.561	242.	282.	-0.11	-1640.
7.0	0.454	0.452	2107.	--	--	--

Table 3. Frequency Response of Negative Capacitance Produced with Feedback Amplifier and Employing $C_f = 0.01$ uf

f (kc)	C_{p1} (uf)	C_{p2} (uf)	R_{p1} (ohm)	R_{p2} (ohm)	$-C_n$ (uf)	$-R_n$ (ohm)	ϕ (degrees)
0.50	0.1812	0.1122	2068.	2055.	-0.0690	+65.4	-0.80
1.00	0.16222	0.09230	3536.	3519.	-0.0699	+ 7.04	-0.18
2.00	0.15739	0.08769	2558.	2585.	-0.0697	- 5.30	+0.18
3.00	0.15658	0.08703	3686.	3868.	-0.0696	- 7.41	0.55
5.00	0.16492	0.09549	1591.	1701.	-0.0694	- 8.63	1.07
10.0	0.15530	0.08565	443.	477.	-0.0697	- 8.39	2.1
20.0	0.13167	0.06186	153.	169.	-0.0700	- 7.97	4.0
30.0	0.1545	0.0852	126.	153.	-0.0701	- 8.14	6.1
50.0	0.1374	0.0689	54.	66.	-0.0701	- 7.10	9.0

Table 4. Frequency Response of Negative Capacitance Produced with Feedback Amplifier and Employing C_f = 0.001 uf

f (kc)	C_{p1} (uf)	C_{p2} (uf)	R_{p1} (ohm)	R_{p2} (ohm)	$-C_n$ (uf)	$-R_n$ (ohm)	ø (degrees)
0.30	0.049	0.041	2614.	2605.	-0.008	+5960.	-5.2
0.50	0.1129	0.1059	2598.	2585.	-0.0071	+3950.	-5.0
1.00	0.13410	0.12706	3850.	3814.	-0.00706	+1250.	-3.2
2.00	0.14450	0.13759	4341.	4300.	-0.00691	+ 292.	-1.45
3.00	0.14142	0.13450	1959.	1951.	-0.00692	+ 124.	-0.92
5.00	0.14327	0.13636	1455.	1454.	-0.00691	+ 10.1	-0.13
10.0	0.1335	0.1265	393.	394.	-0.0070	- 33.4	+0.83
20.0	0.1059	0.0987	128.	129.	-0.0072	- 74.0	+3.8
50.0	0.142	0.132	76.	78.	-0.01	- 34.0	+6.2

Table 5. Locus of the Quantity S (Equation 20) for Negative Capacitance Produced with Feedback Amplifier and Employing $C_f = 0.01$ uf

f (kc)	$-1/C_n$ (10^6df)	$-wR_n$ (10^3df)
0.50	-14.50	205.
1.00	-14.31	44.2
2.00	-14.34	- 66.5
3.00	-14.37	- 139.6
5.00	-14.40	- 271.
10.0	-14.34	- 526.
20.0	-14.30	- 1000.
30.0	-14.27	- 1530.
50.0	-14.27	- 2230.

Table 6. Variation of the Critical Resistance for Oscillation in the Circuit of Fig. 17

C (uf)	R (ohm)
0.00100	0.
0.00400	1.
0.00600	2.
0.00700	1.
0.00800	39.
0.01400	25.

Table 7. Amplifier Parameters and Optimum Element Values

$u_1 \doteq u_2 \doteq 20$

$r_{p1} \doteq r_{p2} \doteq 7700$ ohms

$g_{m1} \doteq g_{m2} \doteq 2600 \times 10^{-6}$ mhos

$R_k \doteq 470$ ohms

$R_L \doteq 120{,}000$ ohms

$R_g \doteq 470{,}000$ ohms

$R_e = 95{,}500$ ohms (calculated)

$K = 0.95 \times 10^{-3}$ mhos (calculated)

$A_m \doteq 8$ (selected)

$R_o \doteq 22{,}000$ ohms (selected to give $A \doteq 8$)

$R_a = 7820$ ohms (calculated)

$A_m = 7.43$ (calculated)

$A_m = 8.35$ (measured)

$R_a = 8800$ ohms (measured)

$C_o + 15$ uuf $= 16$ to 17 uuf (measured, includes 15 uuf voltmeter capacitance)

$C_o = 1$ to 2 uuf (calculated)

$C_s + C_i = 43.8$ (measured)

$C_s = 12.8$ (measured)

$C_i = 31$ uuf (including 15 uuf of VTVM)

$C_f = 6.5$ uuf (calculated optimum value)

$R_c = 15{,}300$ ohms (calculated optimum value)

Table 8. Uncompensated Frequency Response of Two Stage Amplifier (of Fig. 21) with $A_m = 8.35$

f (kc)	E_g (volts)	E_1 (volts)	E_o (volts)	A_1	A_2	$A_1/A_{1\text{-mid}}$
0.020	0.036	1.00	7.7	27.8	7.7	0.290
0.030	0.023	1.00	8.2	43.5	8.2	0.453
0.040	0.018	1.00	8.4	55.5	8.4	0.578
0.055	0.014	1.00	8.3	71.4	8.3	0.744
0.080	0.0125	1.00	8.3	80.0	8.3	0.834
0.100	0.0118	1.00	8.3	84.7	8.3	0.882
0.150	0.0114	1.00	8.4	87.7	8.4	0.914
0.200	0.0109	1.00	8.5	91.7	8.5	0.955
0.300	0.0103	1.00	8.3	97.0	8.3	1.010
0.500	0.0104	1.00	8.4	96.0	8.4	1.000
1.000	0.0107	1.00	8.4	93.5	8.4	0.974
2.00	0.0107	1.00	8.4	93.5	8.4	0.974
3.00	0.0106	1.00	8.4	94.3	8.4	0.982
6.00	0.0105	1.00	8.3	95.2	8.3	0.992
15.0	0.0110	1.00	8.3	91.0	8.3	0.948
25.0	0.0123	1.00	8.3	81.2	8.3	0.846
30.0	0.0131	1.00	8.3	76.3	8.3	0.795
40.0	0.0151	1.00	8.3	66.2	8.3	0.690
50.0	0.0166	1.00	8.3	60.2	8.3	0.627
70.0	0.0205	1.00	8.3	48.8	8.3	0.508
100.	0.028	1.00	8.3	35.7	8.3	0.372
150.	0.039	1.00	8.2	25.6	8.2	0.267
200.	0.051	1.00	8.2	19.6	8.2	0.204

Table 9. Compensated High-Frequency Response with $A_m = 5.2$; $R_c = 0$; $C_f = 5.$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\text{-mid}}$
1.00	0.0105	1.00	95.	1.00
3.00	0.0104	1.00	96.	1.01
10.0	0.0104	1.00	96.	1.01
30.0	0.0101	1.00	99.	1.04
40.0	0.0100	1.00	100.	1.05
60.0	0.0105	1.00	95.	1.00
80.0	0.0123	1.00	81.	0.85
100.	0.0151	1.00	66.	0.69
150.	0.0276	1.00	36.	0.38
200.	0.0425	1.00	24.	0.25

Table 10. Compensated High-Frequency Response with $A_m = 8.35$; $R_c = 15{,}000$ ohms; $C_f = 6.9$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\ mid}$
1.00	0.0105	1.00	95.2	0.96
2.00	0.0105	1.00	95.2	0.96
3.00	0.0103	1.00	97.0	0.98
6.00	0.0101	1.00	99.0	1.00
15.0	0.0102	1.00	98.0	0.99
25.0	0.0103	1.00	97.0	0.98
30.0	0.0101	1.00	99.0	1.00
40.0	0.0098	1.00	102.	1.03
50.0	0.0097	1.00	103.	1.04
70.0	0.0093	1.00	108.	1.08
100.	0.0084	1.00	109.	1.10
150.	0.0064	1.00	156.	1.58
200.	0.0067	1.00	149.	1.50

Table 11. Compensated High-Frequency Response with $A_m = 8.35$; $R_c = 15{,}000$ ohms; $C_f = 5.1$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\ mid}$
25.0	0.0105	1.00	95.2	1.000
30.0	0.0106	1.00	94.3	0.980
40.0	0.0106	1.00	94.3	0.980
50.0	0.0109	1.00	91.7	0.963
70.0	0.0113	1.00	88.5	0.930
100.	0.0116	1.00	86.2	0.905
150.	0.0141	1.00	70.9	0.745
200.	0.0186	1.00	53.7	0.565

Table 12. Compensated High-Frequency Response with $A_m = 8.35$; $R_c = 15{,}000$ ohms; $C_f = 8.6$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\ mid}$
30.0	0.0105	1.00	95.2	1.00
40.0	0.0105	1.00	95.2	1.00
50.0	0.0105	1.00	95.2	1.00
70.0	0.0097	1.00	103.	1.08
100.	0.0084	1.00	119.	1.25
150.	0.0058	1.00	172.	1.81
180.	0.0045	1.00	222.	2.33
200.	0.0049	1.00	204.	2.14

Table 13. Compensated High-Frequency Response with $A_m = 6.9$, $R_c = 6800$ ohms; $C_f = 8.5$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\ mid}$
10.0	0.0090	1.00	111.	1.00
20.0	0.0090	1.00	111.	1.00
30.0	0.0088	1.00	114.	1.03
40.0	0.0084	1.00	119.	1.07
60.0	0.0079	1.00	127.	1.14
100.	0.0062	1.00	161.	1.45
120.	0.0057	1.00	175.	1.58
150.	0.0058	1.00	172.	1.55
200.	0.0095	1.00	105.	0.95

Table 14. Compensated High-Frequency Response with $A_m = 6.9$, $R_c = 6800$ ohms; $C_f = 7.5$ uuf

f (kc)	E_g (volts)	E_1 (volts)	A_1	$A_1/A_{1\ mid}$
1.00	0.0093	1.00	107.	1.00
10.0	0.0090	1.00	111.	1.03
20.0	0.0090	1.00	111.	1.03
30.0	0.0089	1.00	112.	1.05
40.0	0.0087	1.00	115.	1.07
60.0	0.0084	1.00	119.	1.11
100.	0.0078	1.00	128.	1.19
120.	0.0078	1.00	128	1.19
150.	0.0089	1.00	112.	1.05
200	0.0124	1.00	80.6	0.75

Table 15. Measurement of Z_t with Circuit of Fig. 27

f (kc)	E_1 (mv)	E_2 (mv)	E_3 (mv)	$\lvert Z_t \rvert$ (kilohms)
7.00	136.	136.	0.1 - 0.2	20.
8.00	135.	135.	0.2 - 0.3	12.
9.00	135.	135.	0.3	10.
10.0	135.	135.	0.4	7.5
11.0	134.	134.	0.4	7.4
12.0	134.	134.	0.4	7.4
14.0	134.	134.	0.5	5.9
16.0	134.	133.	0.7	4.2
18.0	134.	133.	1.0	2.95
20.0	133.	132.	1.2	2.44
23.0	133.	131.	1.5	1.94
26.0	132.	130.	2.0	1.44
30.0	132.	130.	3.0	0.962
35.0	131.	128.	4.1	0.694
40.0	130.	126.	4.9	0.571
48.0	129.	122.	6.5	0.417
56.0	126.	118.	8.6	0.304
62.0	124.	114.	10.6	0.239
70.0	122.	109.	13.3	0.182

Table 16. Measurement of Z_t with Circuit of Fig. 28

f (kc)	E_1 (mv)	E_2 (mv)	E_3 (mv)	$\lvert Z_t \rvert$ (kilohms)	$\pm \sin \theta$	$\lvert\theta\rvert$ max (degrees)
4.00	264.	264.	0.3	19.5		
6.00	270.	269.	0.75	8.0		
8.00	271.	270.	1.15	5.2		
10.0	278.	276.	1.82	3.36		
13.0	229.	226.	2.52	1.99		
16.0	206.	201.	3.59	1.24	$0.10 + 0.02$	7.
18.0	146.	142.	3.22	0.978	0.00 ∓ 0.02	1.
20.0	152.	148.	3.8	0.865	0.00 ∓ 0.02	1.
25.0	107.	102.	4.8	0.471	0.04 ∓ 0.02	3.
30.0	90.	84.	5.2	0.358	0.20 ∓ 0.04	14.

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